Employee Reload Options: Pricing, Hedging, and Optimal Exercise[1]

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[1]Dybvig, Philip H. and Mark Loewenstein, 2003, Employee Reload Options: Pricing, Hedging, and Optimal Exercise, *Review of Financial Studies* **16**, 145–171.

Abstract

Reload options, call options whose exercise entitles the holder to new options, are compound options that are commonly issued by firms to employees. Although reload options typically involve exercise at many dates, the optimal exercise policy is simple (always exercise when in the money) and surprisingly robust to the assumptions about the underlying stock price and dividend process. As a result, we obtain general reload option valuation formulas that can be evaluated numerically. Furthermore, under the Black-Scholes assumptions with or without continuous dividends, there are even simpler formulas for prices and hedge ratios. In the case when passage of time is required to vest each reload option, no exact valuation formula is yet available, but we provide useful upper and lower bounds.

What Are Reload Options?

Exercise of a reload option gives you (1) a call payoff and (2) new reload options. Exercising a reload option using shares evaluated at the current market price entitles you to one reload option for each share tendered. The new reload options are issued with strike equal to the current stock price and the same maturity as the original reload options. (There is some variation in this; this is the most common pattern.)

Reload options were first offered by Norwest in 1988 and were included in 17% of new stock option plans in 1997, up from 14% in 1996.

More Background

HEMMER, T., S. MATASUNAGA, AND T. SHEVLIN 1996, Optimal Exercise and the Value of Employee Stock Options Granted with a Reload Provision, *Journal of Accounting Research* **36**, 231–255.

- accounting perspective: need to improve on current FASB guidelines
- extensive survey of current practice
- optimal exercise policy and valuation in a binomial model
- introduced reload options to us

ARNASON, S., AND R. JAGANNATHAN 1994, Evaluating Executive Stock Options Using the Binomial Option Pricing Model," working paper, University of Minnesota.

SALY, P. J., R. JAGANNATHAN, AND S. J. HUDDART 1999, "Valuing the Reload Features of Executive Stock Options," *Accounting Horizons* **13**, 219–240.

- optimal exercise policy and valuation in a binomial model

Some Criticisms of Reload Options in the Popular Press

1. Reload options may give the manager essentially unbounded payoff because of the opportunity to reload again and again.
2. Reload options remove from the firm control over the total number of shares it will issue.
3. Reload options give managers an incentive to take on too much risk.
4. Reload options give managers an incentive not to pay big dividends.

These problems are supposed to be particularly bad for reloads with extended maturity, e.g., for which the new options all have 10 years until maturity from time of creation.

Exercise Example: current stock price = $125

- Position before exercise
  - 100 reload options, strike $100, maturity Dec. 31, 2005
- Exercise:
  - tender 80 shares (total market value = $\$125 \times 80 = \$100 \times 100$)
  - tender the 100 original reload options
  - receive 100 shares
  - receive 80 new reload options, strike $125, maturity Dec. 31, 2005
- Position after exercise:
  - 20 shares
  - 80 reload options, strike $125, maturity Dec. 31, 2005

The Effect of Multiple Reloads

strike K, reloads at stock prices $S_1, S_2, \ldots$

For each initial reload option:

- after the first reload, we have $(1 - K/S_1)$ shares, and K/S_1 new reload options, each with strike S_1
- after the second reload, we have $(1 - K/S_1) + (K/S_1)(1 - S_1/S_2) = (1 - K/S_2)$ shares and K/S_2 new reload options, each with strike S_2
- after the ith reload, we have $(1 - K/S_i)$ shares, and K/S_i new reload options, each with strike S_i

No-arbitrage Bounds: Debunking the Criticisms

- The employee never receives net more than one share per reload
  - No-arbitrage upper bound for the value = stock price
  - Strict control on the net number of new shares issued = one per reload option
- No-arbitrage lower bound for the value = American call with the same strike and maturity (since we can always discard the new option)
- Given the properties above, the reload option is not so much different from alternative instruments (stock and options) used in compensation packages, in terms of incentives for taking on risk or paying dividends.
- Note: Throughout we are ignoring taxes and earnings management, which are probably part of the reason for having these options in the first place.

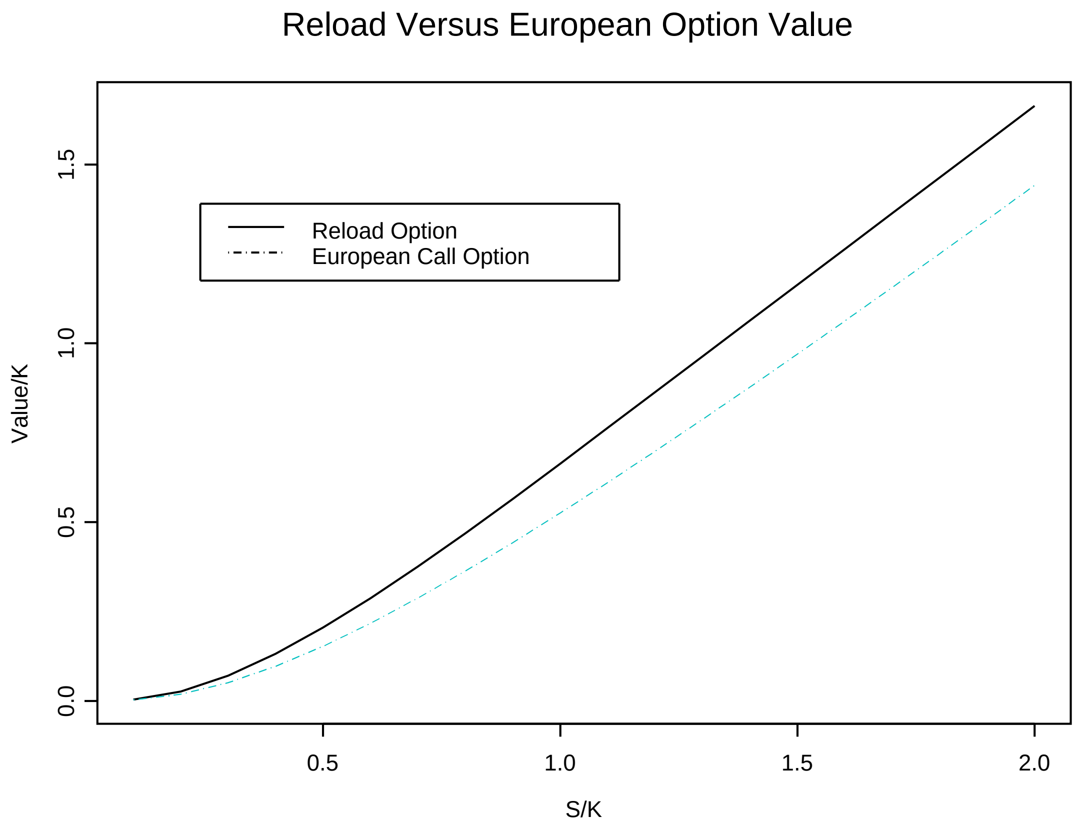

Figure 1: Comparison of reload option value with European call option value

Each reload option is worth a bit more than the corresponding European call option, but not enough more to justify the criticisms in the press.

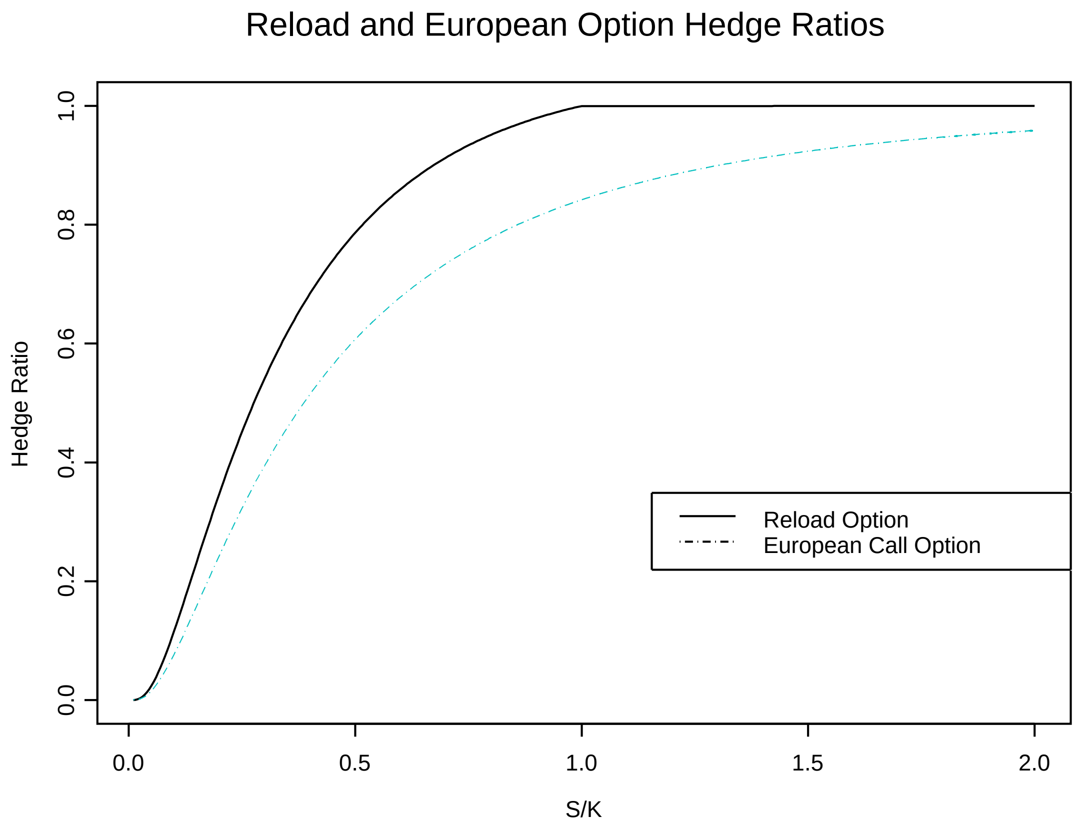

Figure 2: Comparison of reload option hedge ratio with European call option hedge ratio

At a given degree of moneyness, a reload option is more levered than the corresponding European call option.

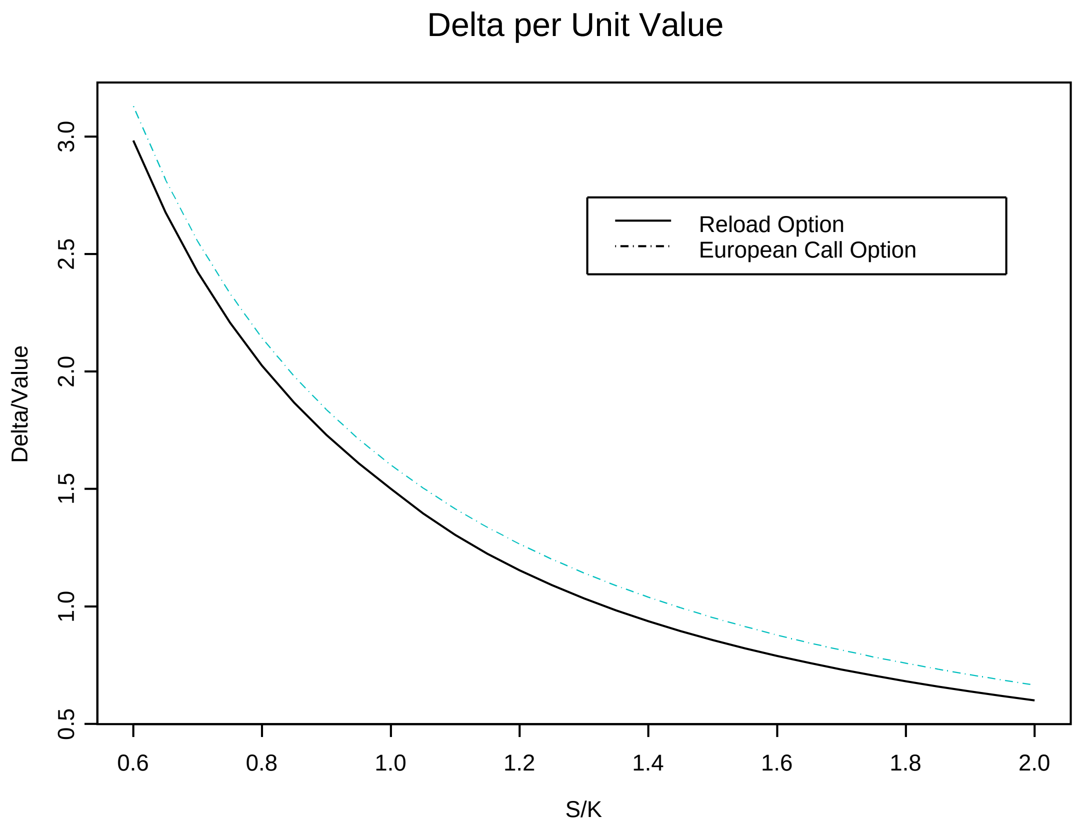

Figure 3: Comparison of reload option delta per unit value with that for a European call option.

However, leverage per unit value is actually a bit less than for the corresponding European call option.

Stock and Bond markets

We use the risk-neutral probabilities (Cox-Ross) and we assume complete frictionless markets. The reinvested locally riskless bond price process is $B(t)$ (nondecreasing and continuous). The stock process is $S(t)$ and there is a corresponding nondecreasing cumulative dividend process $D(t)$. For some results, we will assume that $S(t)+D(t)$ is continuous, which implies that the only jumps in stock prices are downward jumps that correspond exactly to dividend payments.

Notation: $t-$ indicates values before the exercise, if any, at t. (For processes we consider, it also indicates the usual meaning of left limit when that is meaningful.) This notation helps us to include exercise at 0 without separate special treatment.

Valuation: generic

valuation of holding the stock from t to T:

$$(3) \qquad \frac{S(t)}{B(t)} = E_t^*[\frac{S(s)}{B(s)} + \int_t^s \frac{1}{B(u)} dD(u)]$$

time $0-$ valuation of a generic cumulative cash flow C:

$$(4) \qquad E^*[\int_{t=0-}^T \frac{1}{B(t)} dC(t)]$$

Valuation: reload (arbitrary strategy)

strike = K, finitely many reloads:

$$(5) \quad E^*[\frac{S(T)}{B(T)}(1-\frac{K}{X(T)})+\int_0^T(1-\frac{K}{X(t)})\frac{1}{B(t)}dD(t)]$$

where the strike price process $X(t)$ is

$$(6) \quad X(t)=\begin{cases} K & 0- \leq t < \tau_1 \\ S(\tau_1) & \tau_1 \leq t < \tau_2 \\ S(\tau_2) & \tau_2 \leq t < \tau_3 \\ & \vdots \end{cases}$$

and where $\tau_1, \tau_2, \ldots,$ are the excercise dates.

The Optimal Strategy

The value of the reload option depends on the strike price only through $X(t)$, and it is increasing in $X(t)$ for all t. Since exercising whenever the reload option is in the money has uniformly the largest $X(t)$ for each t of all strategies, this must be the optimal strategy. Then the strike price process is

$$(11) \quad M(t) \equiv \max\{K, \max\{S(s); 0 \leq s \leq t\}\}$$

(for the continuous limit), and the value is

$$(12) \quad E^*[\frac{S(T)}{B(T)}(1 - \frac{K}{M(T)}) + \int_0^T \frac{1}{B(t)}(1 - \frac{K}{M(t)})dD(t)].$$

This formula treats the cash flows assuming that any shares gained from tendering were held until T. We have an alternative valuable formula (giving, of course, the same answer) if the proceeds from exercise are liquidated immediately.

Alternative Valuation Formula

(13) $E^*[\int_{0-}^{T} \frac{1}{B(t)} \frac{K}{M(t-)} dM(t)]$,

or

(14) $(S(0)-K)^+ + E^*[\int_{0}^{T} \frac{1}{B(t)} \frac{K}{M(t-)} dM(t)]$.

or, letting $m(t) \equiv \log(M(t)/M(0))$ and invoking continuity of $M(\cdot)$,

(15) $(S(0)-K)^+ + K\, E^*[\int_{0}^{T} \frac{1}{B(t)} dm(t)]$.

or

(16) $(S(0)-K)^+ + K\, (E^*[\frac{1}{B(T)} m(T)] - E^*[\int_{0}^{T} m(t) d\frac{1}{B(t)}])$

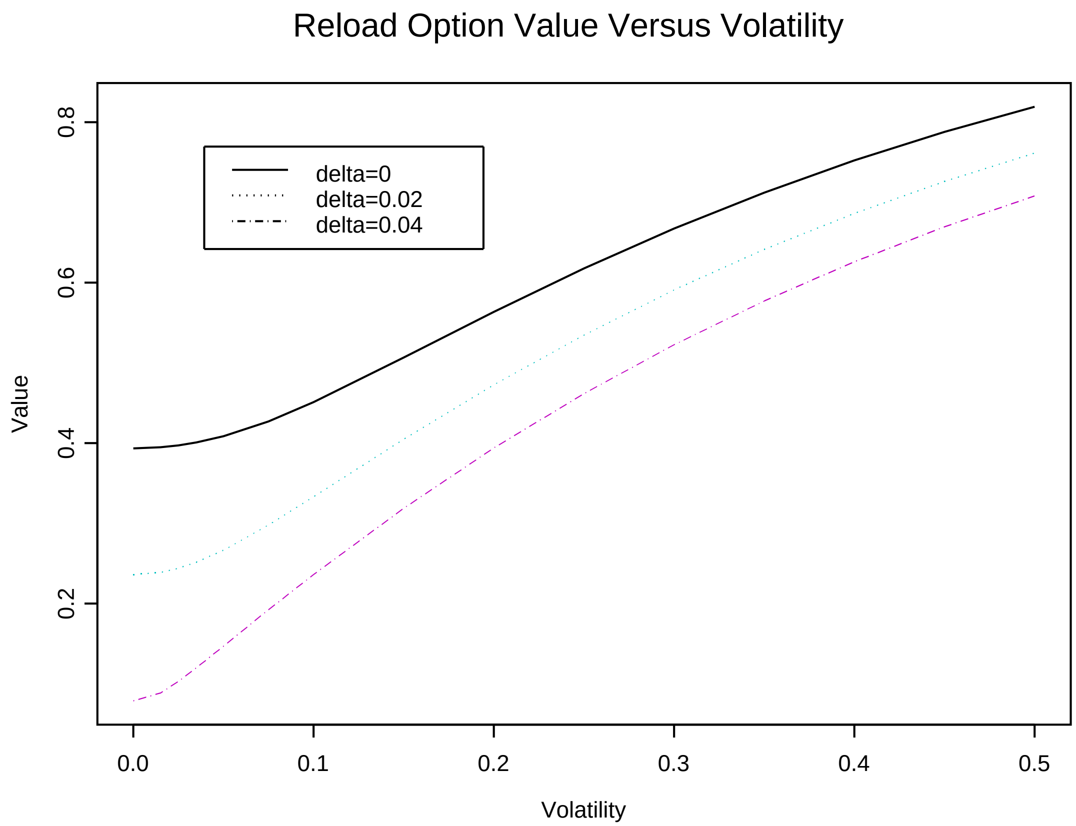

Figure 4: Reload option values for various volatilities and dividend rates

This shows the value of a par reload option with 10 years to maturity and a strike of $1.00 as a function of the volatility (annual standard deviation) for three different annual dividend payout rates (0, 0.2, and 0.4), assuming an annual interest rate of 5%. As for an ordinary call option, the reload's value is increasing in volatility and decreasing in the dividend payout rate.

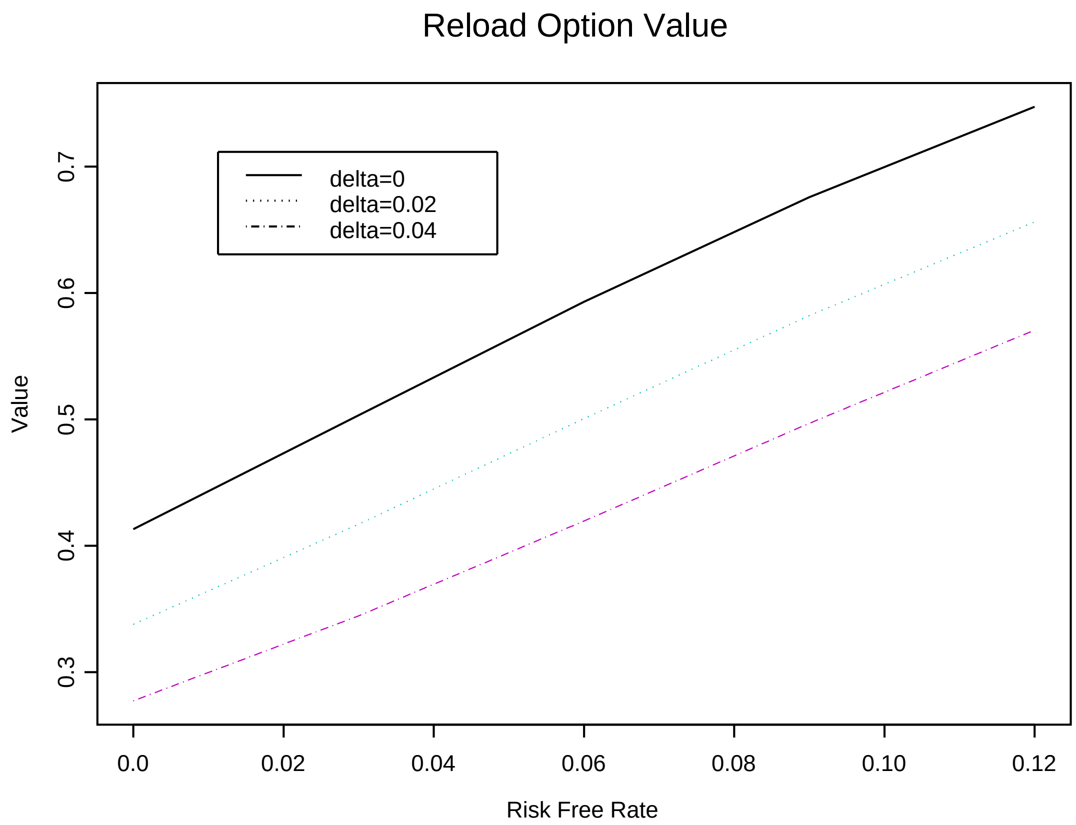

Figure 5: Reload option values for various interest and dividend rates

This shows the value of a par reload option with 10 years to maturity and a strike of $1.00 as a function of the interest rate (annual number) for three different annual dividend payout rates (0, 0.2, and 0.4), assuming an annual standard deviation of .2. As for an ordinary call option, the reload's value is increasing in the interest rate and decreasing in the dividend payout rate.

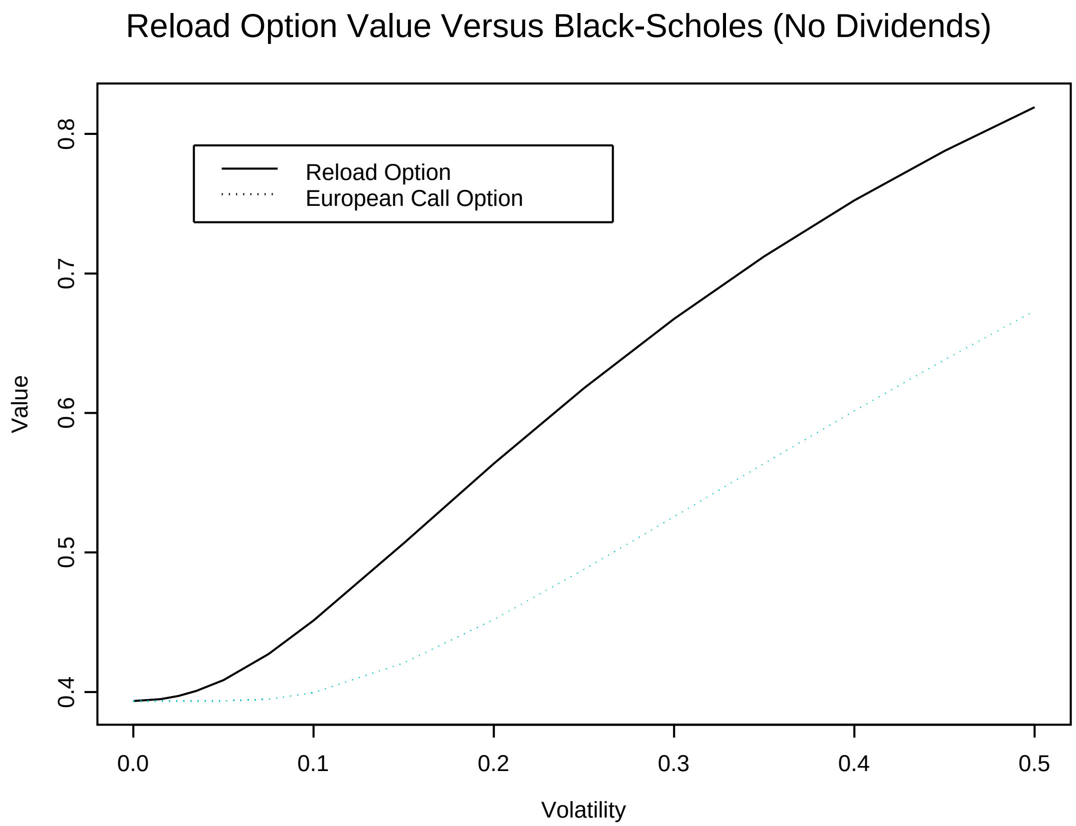

Figure 6: Comparison of reload option values with a Black-Scholes European call option: no dividends

This shows the value of a par reload option (upper curve) and European call (lower curve) with 10 years to maturity and a strike of $1.00 as a function of the volatility (annual standard deviation) when there are no dividends, assuming an annual interest rate of 5%. The two values move further apart as volatilities increase over the range shown, but both asymptote to $1.00 (the stock price) asymptotically.

Assumptions for the Black-Scholes Case with Dividends

(17) $B(t) = e^{rt}$

(18) $S(t) = S(0)\exp((\mu(t) - \frac{\sigma^2}{2} - \delta)dt + \sigma dZ(t))$

(19) $D(t) = \int_0^t \delta S(u)du$

r, δ, σ constant and μ "arbitrary"

Valuation in the Black-Scholes Case with Dividends

Suppose stock and bond returns are given by (17)–(19) and the current stock price is $S(0)$. Consider a reload option with current strike price K and time to maturity τ. Its value is

$$(20) \quad (S(0)-K)^{+}+K(e^{-r\tau}E^{*}[m(\tau)]+r\int_{0}^{\tau}e^{-rt}E^{*}[m(t)]dt)$$

where the cumulative distribution function of $m(t)$ is given by $P^{*}\{m(t)\leq y\}=0$ for $y<0$ and by

$$(21) \quad P^{*}\{m(t)\leq y\}=\Phi(\frac{y-b-\alpha t}{\sigma\sqrt{t}})-exp(\frac{2\alpha(y-b)}{\sigma^{2}})\Phi(\frac{-y+b-\alpha t}{\sigma\sqrt{t}})$$

for $y\geq 0$, where $b\equiv -(\log(K/S(0)))^{+}$, $\alpha\equiv r-\delta-\frac{\sigma^{2}}{2}$ and $\Phi(\cdot)$ is the unit normal cumulative distribution function.

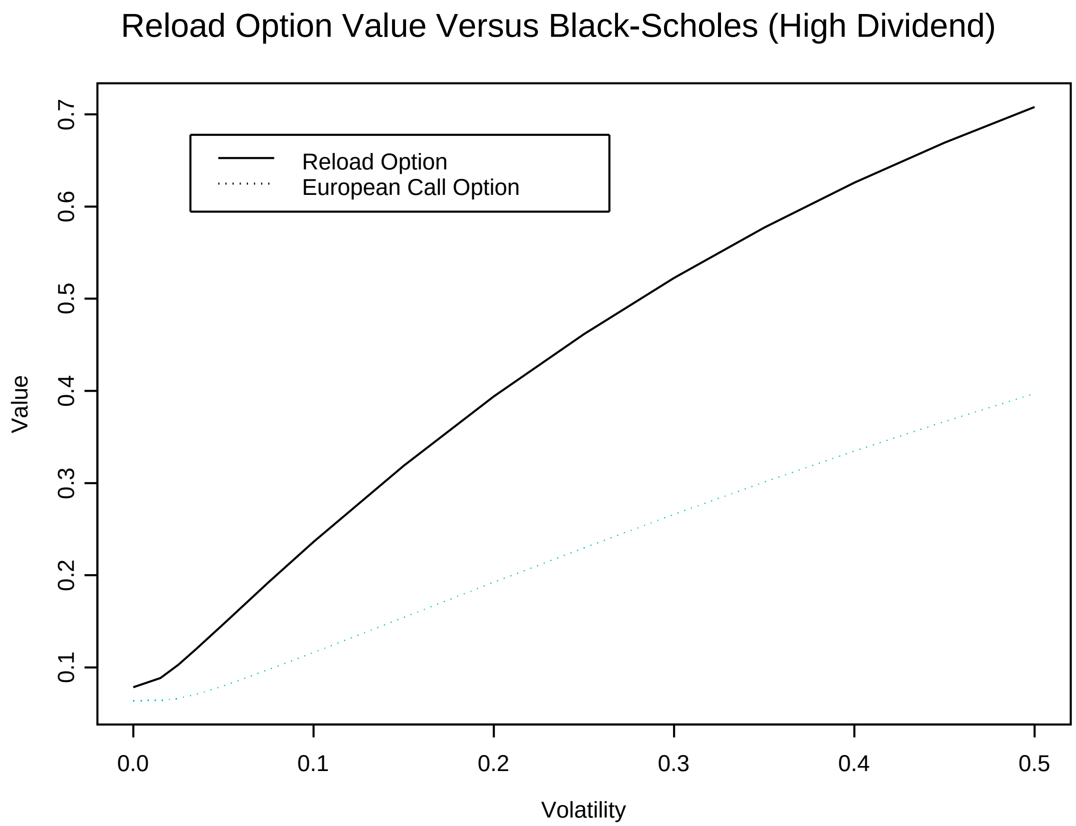

Figure 7: Comparison of reload option values with a Black-Scholes European call option: 4% dividends

This shows the value of a par reload option (upper curve) and European call (lower curve) with 10 years to maturity and a strike of $1.00 as a function of the volatility (annual standard deviation) when there are 4% annual dividends, assuming an annual interest rate of 5%. The two values move further apart more quickly than without dividends as volatilities increase, and in fact the reload asymptotes to a higher value.

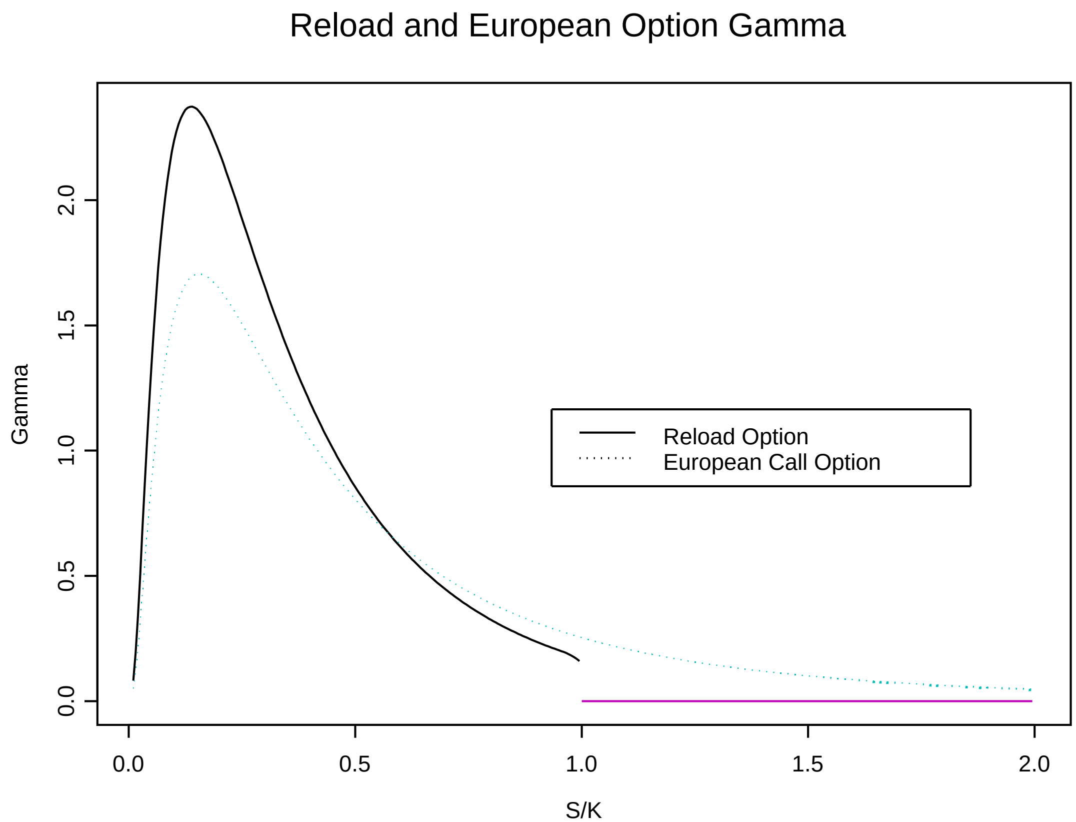

Figure 8: Comparison of reload option gamma and European call option gamma

This shows the gamma of a reload option (dark curve) and a European call option (dashed curve) with 10 years to maturity as a function of S/K when there are no dividends, the annual interest rate is 5%, the strike price is 1, and volatility is 30%.

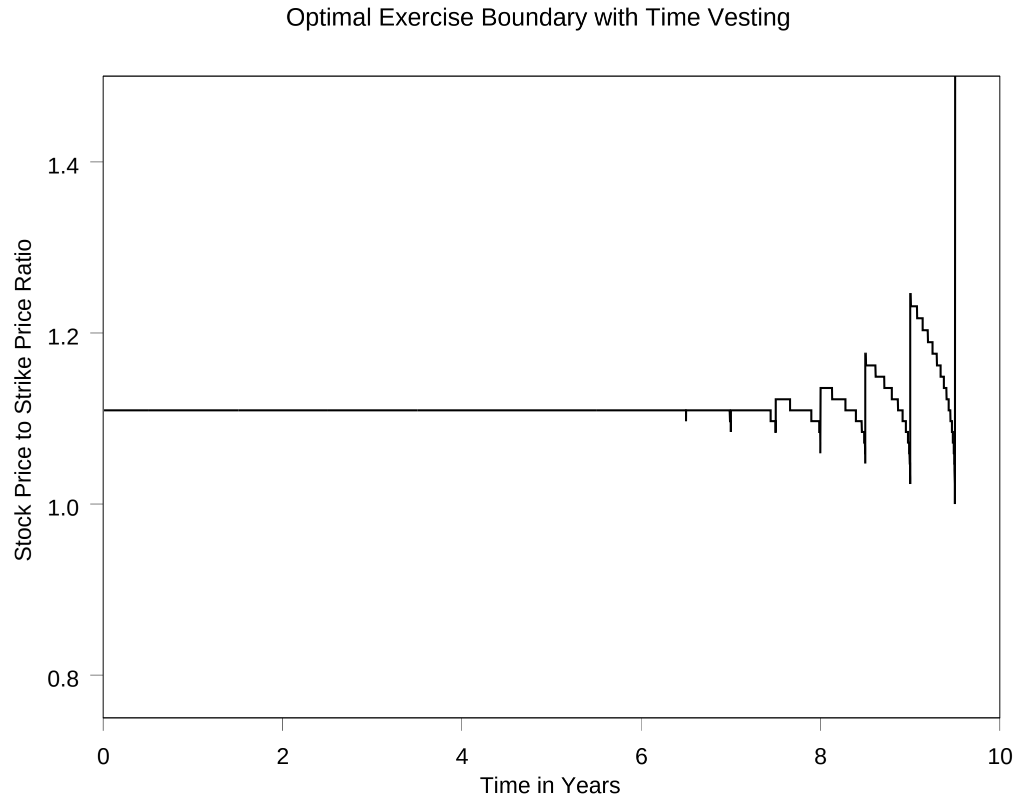

Figure 9: Optimal exercise boundary with time vesting

This optimal exercise boundary for a vested reload option with 10 years to maturity and a six-month vesting period was computed using a trinomial model with 1000 periods per year, assuming an underlying non-dividend-paying stock with standard deviation of 30%/year and an interest rate of 5%/year. The vertical steps are due to the discrete set of possible stock prices. During the last half-year, the option is equivalent to a European call and is never exercised.

Summary of Results

Employee Reload Options:

- Optimal Exercise, quite generally exercise whenever the option is in the money
- Valuation, both general and BS case with dividends
- Optimal Hedge, BS case
- Debunking of wild criticisms
- Sensitivity to vesting