The Cost and Duration of Cash-Balance Pension Plans

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Controversy about the fairness of early transitions from traditional defined-benefit plans to cash-balance plans may have overshadowed the subtleties of funding a cash-balance pension liability. Because crediting rates of cash-balance liabilities float with market rates, the same techniques used to value and hedge floating-rate bonds provide the present value cost and effective duration of a cash-balance liability. The present-value cost of funding a liability varies dramatically across the menu of IRS-sanctioned crediting alternatives. For example, given the yield curve from November 15, 1999, the present value per $1.00 of cash balance of funding a liability paying off 30 years from now varies between $0.90 and $1.48. The effective duration of a cash-balance liability also varies dramatically according to various crediting rates; the effective duration is typically positive but much shorter than the expected time until retirement or other payment and, depending on the choice of crediting rate, can vary by a factor of five or so. These findings are useful for comparing the costs of plans, for comparing how various groups are treated in a plan conversion, or for evaluating the riskiness of any mismatch between assets and liabilities for various funding alternatives.

Cash-balance pension plans have become popular in the United States since first introduced by Bank of America in 1984. These plans are so-called hybrid plans that are regulated as defined-benefit plans (because benefits are based on a formula, not on actual investment results) but resemble defined-contribution plans. In a cash-balance plan, the cash balance is the current lump-sum accrued pension benefit.[1] An employee's cash balance comes from periodic "pay-related credits" linked to salary and wages at a rate that usually depends on age and seniority and from "interest-related credits" at an annually adjusted crediting rate linked to a market rate (usually a constant-maturity bond yield or discount) or the U.S. Consumer Price Index (CPI). Cash-balance plans are easy for employees to understand because the accumulation of cash balance works the same as the accumulation of the balance in a savings account at a bank or thrift institution.

The consulting company PwC Kwasha HR Solutions estimates that between 400 and 1,000 employers have some sort of cash-balance plan in place (see Anand 1999). A short list of employers that have adopted or intend to adopt cash-balance plans includes Ameritech, AT&T, Avon Products, Bell Atlantic, Bell South Corporation, Carolina Power & Light, California State Teachers' Retirement System, Citigroup, Chemical Bank, First Chicago NBD Corporation, Georgia-Pacific Corporation, IBM Corporation, Niagara Mohawk Power Corporation, Public Service Electric and Gas Company, SBC Communications, and the World Bank.

In general, cash-balance plans are user-friendly because employees who switch jobs can take benefits with them when they leave and because it is easy for the sponsor to communicate to employees what benefits have accrued. Many early transitions to cash-balance plans, however, have generated public controversy. The controversy usually centers around whether workers in different age groups have been treated fairly in the transition to the plan.

Whether or not existing transitions are fair, there remains the interesting question we address

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here of evaluating the cost and effective duration of cash-balance liabilities. Because a cash-balance plan liability is, in effect, a sort of floating-rate bond, modern term-structure theory can be used to value the liability and compute its effective duration. The unit of analysis we use is an ideal unit of cash-balance liability to be withdrawn (presumably because of retirement or departure from the plan) at a known maturity date in the future.[2] The cost of funding this unit of liability depends on the current slope of the term structure of interest rates, any margin associated with various crediting assets sanctioned by the U.S. IRS, time to maturity, and (to much less extent) the volatility of interest rates.

Although a mismatch between cost and cash balance is most pronounced when yield curves are very steep or inverted, a significant discrepancy may exist even for a yield curve that is relatively flat with a hump. For example, as we calculate later using a "benchmark" yield curve from November 15, 1999, the market-value cost of a cash-balance liability maturing in 20 years and crediting at the 30-year par U.S. Treasury yield is $0.92 per dollar of cash balance, which amounts to –41 basis points (bps) a year. The market-value cost of funding this liability if it credits at the 10-year yield is about $0.97 per dollar of cash balance (–15 bps/year), and the market-value cost of funding this liability if it credits at the 1-year rate plus the IRS guideline margin is $1.18 per dollar of cash balance (83 bps/year). Because the IRS guidelines permit crediting at the 10-year rate without any margin, to credit at the 1-year rate plus the IRS margin, rather than at the 10-year rate, costs the plan sponsor about 100 bps a year [83 – (–15)]. Interestingly, nonzero IRS guideline margins tend to dominate the other effects and make cash-balance liabilities expensive to fund.

The duration of a cash-balance liability depends primarily on the maturity of the liability and the mismatch between the crediting reset interval and the duration of the crediting asset. For annual resets that use the one-year Treasury rate, the duration should be close to zero (i.e., the claim's value today is almost insensitive to interest rate movements). The reason is that increases in the interest rates used to discount the future payments are offset one-for-one by increases in the projected interest-crediting rates determining the liability. In the benchmark case, with a 20-year liability credited at the 1-year Treasury rate, the computed duration is 0.428 years; it is not exactly zero only because the computations assume quarterly compounding of interest-related credits (annual compounding would make the computed duration exactly zero).

In contrast, a cash-balance liability with a lump-sum payment in 20 years credited at the 30-year par Treasury yield has an expected duration of about 4 1/2 years. The duration is expected to be significantly less than 20 years (the duration of a fixed claim, such as a Treasury strip maturing in 20 years) because significant cancellation characterizes movements in rates used for crediting and discounting. The cancellation is incomplete, however, because the 30-year Treasury yield moves less than one-for-one with changes in the appropriate discount rate. In other words, increases in the interest rates used to discount the future payments are offset only partially by increases in the projected crediting rates that determine the liability.

The extent to which the increase in future crediting rates offsets the increase in the discount factor depends critically on the time to maturity of the crediting asset and the maturity of the liability. The extent of the offset is also related to historical knowledge about how interest rates are related. For example, the increase in the projected future 10-year bond yields over the next 10 years following an upward shift in the term structure of interest rates is greater than the expected increase in the projected future 30-year bond yields over the same 10 years. For this reason, given the benchmark yield curve, the effective duration of a liability accruing at the 10-year rate is only about 2 1/3 years whereas the effective duration of a liability accruing at the 30-year rate is about 4 1/2 years.

The analysis here does not try to include the impact of taxes or liquidity on a company's valuation of different plans. In other words, valuation treats the pension liabilities as fully liquid cash flows to be funded by liquid assets and computes the value to a tax-exempt investor. In fact, pension liabilities are illiquid, and it should be possible to fund them more cheaply with illiquid investments. Unfortunately, accepted asset-pricing models do not quantify the liquidity discount.[3] A sponsor's valuation of a cash-balance plan liability may also be influenced by the sponsor's changing tax status over time and the sponsor's capital constraints (implying an internal liquidity premium). Again, accepted models do not provide a good way to quantify these effects.

Definitions and an Example

The unit of analysis is the market-value cost (i.e., the appropriate present value at market rates of interest) of a liability at a fixed maturity represented by $1.00 of cash balance today. This cost is the appropriate present value of the final payoff computed by projecting only future interest-related credits and no pay-related credits. This cost is the fair-market-value analog of the projected benefit obligation (PBO) in traditional defined-benefit plans. In a traditional plan, the PBO, an accounting measure of the value of the benefits that have accrued so far, projects the lump-sum pension benefit based on current years of service and a projected salary at retirement and discounts this value at a corporate bond rate.[4] The specific calculations in this article assume that when the employee will retire or leave the company is known for certain, but these calculations could be merged with an actuarial analysis of departure rates to correspond more closely to traditional PBO calculations (as in Kopp and Sher 1998).

Our analysis varies time to retirement, the market rate underlying the crediting rate, and the margin used in computing the crediting rate.[5] The crediting rate is a prespecified market rate, a prespecified market rate plus a margin, or the CPI plus a margin. For example, the crediting rate for the California State Teachers Retirement System plan is the average yield on 30-year Treasury notes during the preceding year. In principle, a more complicated nonlinear rule for the crediting rate or a rule based on the average of several rates could be used, but IRS "safe-harbor" provisions are likely to induce companies to choose among the simple rules analyzed here.

Table 1 shows the IRS suggestions for margins to be used with various crediting rates.[6] A company following these suggestions has a safe harbor from the default rules for minimum lump-sum contributions; without the safe harbor, the company bears risk through the "whipsaw" effect—wild fluctuation of the benefit to be paid to workers who leave the plan (see Coleman 1998). The guidelines allow a company to credit at the 3-month Treasury discount rate plus 175 bps, at the 1-year Treasury yield plus 100 bps, or at the 30-year Treasury yield flat. Companies could also offer employees an inflation hedge by crediting at the CPI plus 300 bps. The option is attractive for companies that believe the real rates on inflation-linked Treasuries (currently around 3 1/2 percent) will be sustained.

Practice seems to focus on crediting at a rate given by the guidelines (perhaps for regulatory reasons not covered here), but it is interesting to note that IRS Notice 96-8 says that the crediting rate is to be no greater than one of the rates given by a guideline. It may seem counterintuitive that the IRS is protecting employees by limiting the return on their pensions. After all, given today's cash balance, having a higher crediting rate implies more money for the worker either at retirement or at the time of an early departure. The rule that the IRS is enforcing, however, is meant to limit the loss of pension for workers who leave after an initial vesting period but before retirement. In other words, if one takes as given the projected benefit at retirement, using a higher crediting rate implies a lower initial cash balance and tends to penalize workers who leave the company early, which disqualifies the plan for special tax considerations. Crediting at a rate less than one given by the guidelines is arguably less expensive for the plan sponsor for a given level of employee satisfaction because communicating to employees the value of a higher income-related credit may be easier than communicating the value of a higher interest-related credit.

Table 1. IRS Suggested Guidelines for Crediting Rates

Quantity	Standard Index	Associated Margin
Discount rate	3-month T-bills	175 bps
Discount rate	6- or 12-month T-bills	150
Yield	1-year Treasuries	100
Yield	1- or 3-year Treasuries	50
Yield	5- or 7-year Treasuries	25
Yield	10-year or longer Treasuries	0
Rate of change	CPI-U (urban cost of living)	300

Source: Data from IRS Notice 96-8, Internal Revenue Bulletin (1996-6):23–26.

The following example illustrates the cost and effective duration of two cash-balance liabilities, ignoring the complexities associated with uncertain future interest rates and compounding. The example compares two cash-balance liabilities with different crediting rates but the same current cash balance, B_0, and time to exit of four years. Liability A credits at the two-year zero-coupon bond rate, and Liability B credits at the three-year zero-coupon bond rate.

The cost of the liability depends on anticipated future interest rates. Consider the forward rates implied by the current term structure as forecasts of future interest rates. With no uncertainty, these forward rates would be perfect projections (or else arbitrage would exist). Denote by $z_1, z_2, z_3, \ldots, z_n$ the zero-coupon interest rates quoted now at time 0 for Treasury strips maturing, respectively, one, two, three, through n years from now. Then, the one-year forward rates, $f_1, f_2, f_3, \ldots, f_n$, implicit in today's yield curve for maturities one, two, three, and n years from now are given approximately by solving $z_1 = f_1$, $z_2 = (f_1 + f_2)/2$, $z_3 = (f_1 + f_2 + f_3)/3$, $\ldots$, $z_n = (f_1 + f_2 + f_3 + \ldots + f_n)/n$, for $f_1, \ldots, f_n$.[7] Therefore, $f_1 = z_1$, $f_2 = 2z_2 - z_1$, $f_3 = 3z_3 - 2z_2$, $\ldots$, $f_n = nz_n - (n-1)z_{n-1}$.

The future crediting rates are inferred from the forward rates. Recall that Liability A credits at the two-year zero-coupon yield and Liability B credits at the three-year zero-coupon yield. Then, the crediting rate is $r_t^c = (f_t + f_{t+1})/2$ in Liability A and $r_t^c = (f_t + f_{t+1} + f_{t+2})/3$ in Liability B. Therefore, the terminal cash balance in Liability A is

$$B_4^A = B_0\left(1 + \frac{f_1 + f_2}{2} + \frac{f_2 + f_3}{2} + \frac{f_3 + f_4}{2} + \frac{f_4 + f_5}{2}\right)$$
$$= B_0\left(1 + \frac{f_1}{2} + f_2 + f_3 + f_4 + \frac{f_5}{2}\right) \quad (1)$$

and the terminal cash balance in Liability B is

$$B_4^B = B_0\left(1 + \frac{f_1 + f_2 + f_3}{3} + \frac{f_2 + f_3 + f_4}{3} + \frac{f_3 + f_4 + f_5}{3} + \frac{f_4 + f_5 + f_6}{3}\right)$$
$$= B_0\left(1 + \frac{f_1}{3} + \frac{2f_2}{3} + f_3 + f_4 + \frac{2f_5}{3} + \frac{f_6}{3}\right). \quad (2)$$

The cost of the liability is the present value of the projected retirement balance, which is discounted back at the rates implicit in the yield curve. For Liability A, which credits at the two-year zero-coupon rate, the cost is

$$C_0^A = B_0\frac{1 + (f_1/2) + f_2 + f_3 + f_4 + (f_5/2)}{1 + f_1 + f_2 + f_3 + f_4}$$
$$\approx B_0\left[1 + \left(\frac{f_1}{2} + f_2 + f_3 + f_4 + \frac{f_5}{2}\right) - (f_1 + f_2 + f_3 + f_4)\right] \quad (3)$$
$$= B_0\left[1 + \frac{1}{2}(f_5 - f_1)\right].$$

Similarly, for Liability B, which credits at the three-year zero-coupon rate, the cost is

$$C_0^B = B_0\left[\left(1 + \frac{f_1}{3} + \frac{2f_2}{3} + f_3 + f_4 + \frac{2f_5}{3} + \frac{f_6}{3}\right) - (f_1 + f_2 + f_3 + f_4)\right] \quad (4)$$
$$= B_0\left[1 + \frac{2}{3}(f_5 - f_2) + \frac{1}{3}(f_6 - f_1)\right].$$

The deviation of the cost of the liability from the cash balance in the example depends on the slope of the yield curve and is the result of a mismatch between the duration of the crediting asset and the interval between crediting rate resets. If the yield curve is flat or if the crediting rate is the one-year yield, the cost of the liability equals the cash balance. When the maturity of the crediting asset is greater than one year, the cost per unit of cash balance is 1 plus a weighted sum of differences between forward rates at long maturities and forward rates at short maturities. Given an upwardly sloping yield curve, accrual to the end is at higher rates than discounting back to the present and the cost exceeds the cash balance. The opposite is true in a downwardly sloping yield curve, and the cost is less than the cash balance. These effects become more pronounced as the mismatch between the crediting period and the maturity of the crediting asset increases, as can be seen from comparing the formulas for liabilities crediting at two-year and three-year maturity bond rates. In a humped yield curve, one like the benchmark yield curve used in the more exact computations in the section "Computed Cost and Effective Duration of Liabilities," the sign of the effect will vary with the length of the program and the duration of the crediting asset.

To see how, consider the two sample yield curves in **Table 2**, which are upwardly sloping with different slopes. Starting from the currently quoted discount bond yields, forward rates and the two crediting rates are computed as described previously.

With an upwardly sloping term structure and a crediting-asset maturity greater than the one-year spacing between resets, the rates at which the cash balance credits are greater than the corresponding one-year forward rates, which are the one-year returns that can be locked in today. Consequently, the cash balance grows faster than the corresponding rate at which future cash flows are discounted and the value of the liability is larger than the cash balance. The amount by which the crediting rate exceeds the corresponding spot rate depends most significantly on any margin (none here), the slope of the yield curve (larger in Panel B of Table 2), and the duration of the bond underlying the crediting rate (larger for the three-year bond than for the two-year bond).

This example can also be used to highlight how the relative cost of liabilities depends on any margins added. In the upwardly sloping yield curve in Panel A, funding a liability that credits at the two-year rate would be cheaper unless the margin to be added to the two-year yield is more than 0.20 percent higher than the margin added to the three-year rate. With the steeper yield curve in Panel B, funding a liability that credits at the two-year rate would be cheaper unless the margin added to the two-year yield is more than 0.50 percent higher than the margin added to the three-year rate. Note also that when the yield curve is upwardly sloped, positive margins may be unappealing from the standpoint

Table 2. Numerical Example

Years Out	Zero-Coupon Yield Now	Forward Rate	Two-Year Crediting Rate	Three-Year Crediting Rate
A. Upwardly sloping yield curve				
1	5.0%	5.0%	5.2%	5.4%
2	5.2	5.4	5.6	5.8
3	5.4	5.8	6.0	6.2
4	5.6	6.2	6.4	6.6
5	5.8	6.6	na	na
6	6.0	7.0	na	na
B. Steeply upwardly sloping yield curve				
1	5.0%	5.0%	5.5%	6.0%
2	5.5	6.0	6.5	7.0
3	6.0	7.0	7.5	8.0
4	6.5	8.0	8.5	9.0
5	7.0	9.0	na	na
6	7.5	10.0	na	na

na = not applicable.

of the plan provider, in that even without margins, the cash balance grows faster than the bond returns.

The factors that determine the effective duration of a cash-balance liability can also be seen from Equations 3 and 4. The effective duration of a cash-balance liability (or any other cash flow stream) is defined to be the maturity of a zero-coupon bond (such as a Treasury strip) that has the same sensitivity as the cost to a shock in interest rates. A parallel shift in the term structure of interest rates implies that all spot rates (hence, all forward rates) change by the same amount. In this case, the cost of the liability is clearly invariant to interest rate changes and the effective duration is zero. The change in the expected accrual rates resulting from a rate change is exactly offset by the change in the discount factor, at least for a parallel shift in the yield curve.

A more realistic assumption is that a shock to interest rates will have less impact on longer term rates than on shorter term rates. For example, the empirical estimation in Dybvig (1997) suggests that an interest rate shock loses about 15 percent of its impact for each year (1.26 percent each month) that one moves out the forward rate curve. Market participants are likely to react to news today by making large changes in their views about interest rates over the next year, but they are less likely to make large changes in their views about interest rates 25 years from now.[8]

The normal pattern is that an event that results in large increases in forward rates one and two years out, f_1 and f_2, will generate smaller increases in forward rates four and five years out, f_4 and f_5. This pattern will cause the cost of the liability that accrues at the three-year rate to go down, as can be seen from Equation 4, which implies that the effective duration is greater than zero because the cost moves in the same direction as a bond price would. The duration is much less than the maturity of the liability, however, because of the cancellation of rates in the derivation of the pricing formulas. For example, a zero-coupon bond with two years to maturity, which by definition has a duration of 2, would have a price of about $B_0(1 - f_1 - f_2)$ and would be more sensitive to an interest rate move than either cash-balance liability maturing in four years. It is also clear from Equations 3 and 4 that Liability B, which credits at the three-year rate, is more sensitive to interest rates and, therefore, has a higher effective duration than the liability that accrues at the two-year rate.

Computed Cost and Effective Duration of Liabilities

In this section, we evaluate the cost of funding a cash-balance liability when given a particular benchmark yield curve. Analysis here includes all the underlying rates in Table 1 except the CPI.[9] For each underlying rate, both the cost and the effective duration are computed (1) for no margin ("flat") and for the margin under the IRS guideline, (2) for three liability maturities (10 years, 20 years, and 30 years), and (3) for both random and nonrandom interest rate models. Each number computed using the random interest rate model is based on Monte

Carlo simulation with 200,000 random interest rate paths, which implies a simulation error of no more than a few basis points.

The model of interest rates used for computing the terminal balance and discounting back to the present is critical for computing the cost. The analysis is based on one of two interest rate models consistent with a benchmark Treasury strip curve traded for settlement on November 15, 1999 (a convenient date because the strips mature at equal three-month intervals from that date). The simpler model is a certainty model, which forecasts that future interest rates will be equal (for certain) to a smoothed and extrapolated version of the implied forward rates.[10] This certainty model resembles the analysis of the example in the previous section, but it includes proper accounting for compounding and other institutional details. The more sophisticated model describes uncertainty in interest rates using a mean-reverting Vasicek (1977) Gaussian model of interest rates in which the mean rates are assumed to vary over time in a way that replicates the smoothed and extrapolated version of the yield curve on November 15, 1999. The standard deviation of interest rates is taken to be 1 percent a year, and the mean reversion in the model is taken to be 15 percent a year, consistent with actual observed yield curves.[11]

The calculations of cost and effective duration are shown in **Table 3** and **Table 4**. Table 3 gives the results from the sophisticated model with random interest rates, and Table 4 gives the results from the simpler certainty model. Within each table, the A panel gives the cost when crediting at a given rate without any margin and the B panel gives the cost when crediting at the rate plus the IRS margin given in Table 1. Within each panel, the maturity of the program is given in the top row and the base crediting rate is given in the left column. The numbers in the table depend on the specific yield curve. The numbers would also rely on implementation details such as the frequency of compounding or

Table 3. Market-Value Cost and Effective Duration of Cash-Balance Liabilities: Random Model with and without Margins Added to the Market Rates

	10-Year Maturity		20-Year Maturity		30-Year Maturity	
Specification	Cost	Effective Duration	Cost	Effective Duration	Cost	Effective Duration
A. No margins						
3-Month discount	0.962	0.599	0.930	1.046	0.905	1.321
6-Month discount	0.959	0.754	0.923	1.315	0.894	1.664
12-Month discount	0.955	1.061	0.907	1.847	0.871	2.342
1-Year yield	0.990	0.247	0.979	0.428	0.971	0.537
2-Year yield	0.998	0.399	0.983	0.678	0.973	0.858
3-Year yield	1.004	0.546	0.985	0.921	0.973	1.169
5-Year yield	1.013	0.822	0.985	1.376	0.972	1.758
7-Year yield	1.018	1.072	0.982	1.791	0.968	2.299
10-Year yield	1.020	1.395	0.973	2.338	0.958	3.025
20-Year yield	1.003	2.113	0.939	3.650	0.923	4.823
30-Year yield	0.989	2.542	0.920	4.491	0.904	6.003
B. IRS margins						
3-Month discount	1.133	0.727	1.290	1.266	1.479	1.606
6-Month discount	1.104	0.863	1.222	1.503	1.362	1.907
12-Month discount	1.099	1.166	1.202	2.030	1.329	2.582
1-Year yield	1.087	0.323	1.181	0.557	1.286	0.704
2-Year yield	1.045	0.437	1.079	0.742	1.120	0.941
3-Year yield	1.052	0.584	1.082	0.985	1.120	1.252
5-Year yield	1.037	0.841	1.033	1.408	1.043	1.799
7-Year yield	1.042	1.090	1.029	1.822	1.038	2.340
10-Year yield	1.020	1.395	0.973	2.338	0.958	3.025
20-Year yield	1.003	2.113	0.939	3.650	0.923	4.823
30-Year yield	0.989	2.542	0.920	4.491	0.904	6.003

Notes: Calculations are for liabilities with known maturity and no early departure before maturity. The cost is per dollar of cash balance and can deviate significantly from par.

Table 4. Market-Value Cost and Effective Duration of Cash-Balance Liabilities: Certainty Model with and without Margins Added to the Market Rates

	10-Year Maturity		20-Year Maturity		30-Year Maturity	
Specification	Cost	Effective Duration	Cost	Effective Duration	Cost	Effective Duration
A. No margins						
3-Month discount	0.963	0.596	0.935	1.038	0.913	1.305
6-Month discount	0.961	0.752	0.928	1.307	0.902	1.647
12-Month discount	0.956	1.059	0.913	1.838	0.880	2.325
1-Year yield	0.991	0.246	0.981	0.423	0.974	0.530
2-Year yield	0.998	0.398	0.984	0.674	0.974	0.851
3-Year yield	1.004	0.545	0.985	0.917	0.974	1.163
5-Year yield	1.012	0.821	0.984	1.372	0.972	1.752
7-Year yield	1.017	1.070	0.981	1.787	0.967	2.295
10-Year yield	1.019	1.394	0.971	2.335	0.957	3.021
20-Year yield	1.002	2.115	0.937	3.651	0.922	4.824
30-Year yield	0.988	2.546	0.918	4.495	0.903	6.007
B. IRS margins						
3-Month discount	1.135	0.726	1.297	1.259	1.493	1.592
6-Month discount	1.106	0.861	1.228	1.495	1.375	1.892
12-Month discount	1.101	1.165	1.209	2.023	1.343	2.567
1-Year yield	1.088	0.322	1.183	0.553	1.290	0.697
2-Year yield	1.046	0.436	1.080	0.738	1.122	0.934
3-Year yield	1.052	0.582	1.082	0.980	1.122	1.246
5-Year yield	1.036	0.839	1.032	1.404	1.043	1.793
7-Year yield	1.041	1.088	1.028	1.818	1.038	2.335
10-Year yield	1.019	1.394	0.971	2.335	0.957	3.021
20-Year yield	1.002	2.115	0.937	3.651	0.922	4.824
30-Year yield	0.988	2.546	0.918	4.495	0.903	6.007

Note: See notes to Table 3.

whether crediting is at the average rate over the previous year or at the rate at the start of the year. Our main conclusions do not seem to depend materially on those details.

IRS Margins: Very Expensive. Perhaps the most dramatic pattern in Tables 3 and 4 is the impact of the IRS margins. Table 1 gives no margins for crediting at 10-, 20-, or 30-year rates, but for all other underlying rates, the IRS margin has the dominant effect on cost. For example, the cost per dollar of cash balance in the IRS case for 20-year maturity and crediting at the 12-month discount rate (Table 3, Panel B) is $1.202. The extra cost of $0.202 corresponds to a compounded return of 92.42 bps a year, which can be viewed as the excess yield of the liability over the yield of Treasuries of the same effective duration. This excess is attributable almost entirely to the 100-bp margin in the IRS rules.

One way to think of the impact of the IRS guidelines is to ask how much the slope of the yield curve would have to change to overcome the impact of the margin. Under the IRS guidelines, crediting based on the 1-year Treasury yield is associated with a margin of 100 bps but crediting at the 30-year Treasury yield has no associated margin. In the benchmark case, given in Panel B of Table 3, with 20 years to maturity, the cost of the liability is $1.181 with crediting at the 1-year yield or $0.920 with crediting at the 30-year yield. The difference in cost is $0.261. To compute how much to change the slope of the yield curve to make the two crediting rates equally costly, the rule of thumb we develop later (Equations 14 and 15) can be used. Based on this rule, the cost under the 1-year yield does not vary with the slope of the yield curve (because there is no mismatch between the maturity of the crediting asset and the reset period). If we take the duration of a 30-year bond to be 9 years (this exact value is not critical), the required change in the difference between the 20-year forward rate and the spot rate is (by Equation 14) equal to a whopping 6.5 percent—that is, 0.261/[(9 – 1)/2].

A plan sponsor therefore has strong incentives to choose a crediting alternative without any margin (if the sponsor is choosing according to the guidelines) even if the sponsor believes that investment performance can beat the margin in the IRS guidelines. For example, by taking on some acceptably small amount of credit risk, a plan sponsor may think achieving the 1-year Treasury yield plus 100 bps will be easy. By crediting at the 10-year rate, however, the plan sponsor could take on the same credit risk, lengthen the portfolio a bit to realign the duration, and collect the same 100 bps to reduce future contributions. Conceptually, a company might be able to explain to workers the benefit of a short-maturity crediting rate and the corresponding margin, but it is more likely that whatever value is added for the workers will not be appreciated.

The dominant impact of the IRS margins could, in principle, be overcome by other factors in extreme cases—for example, when the yield curve is sloping upward very steeply. However, this sort of temporary condition should not usually affect the ranking of crediting rate alternatives. Although the yield curve today may make choosing an IRS option with a margin for contributions attractive today, the yield curve will change next year, in 5 years, and in 10 years, and the plan sponsor is likely to lose more on future contributions and new employees than is gained on current plan members' current cash balances. The only possible exception is in a plan of inactive participants without ongoing contributions; even so, taking on a margin will rarely be optimal.

From a social policy perspective, IRS guidelines that encourage crediting at long rates may not be a good idea. Even if yields at the long end are expected to be higher than yields at the short end, crediting at the long rate is riskier in real terms for plan participants because short rates track inflation better than long rates. Furthermore, crediting at a long rate is riskier for the Pension Benefit Guarantee Corporation (PBGC). A fund crediting at a long rate and using the duration hedge is subject to a sort of reverse whipsaw effect, namely, that the underlying investments will move around more than the cash balance itself. If the rates go up and the assets' value goes down, the plan may not have enough assets to liquidate to cover a large number of departures. This possibility is a problem for the pension insurance fund only if it happens at a time when the company is unable to make up the difference, but the effect may occur broadly in the economy in a time of high interest rates and high unemployment, thereby creating big problems for the PBGC's insurance fund.

Because the CPI is not a market rate, the 300-bp margin endorsed by the IRS is not necessarily excessively costly. Crediting at the CPI plus 300 bps may be appealing to workers (because the final payment is not risky in real terms) and also to employers (because the margin is less than at least the current yield on indexed Treasuries). Computing the cost and effective duration for a cash-balance liability credited at CPI plus 300 bps is more difficult than for the other cases and necessarily depends significantly on a view of what inflation will be and how inflation will relate to interest rates.

Determinants of Effective Duration. The cash-balance pension liability's effective duration is typically positive but smaller than the maturity of the liability or the duration of the asset underlying the crediting rate. The specification of a cash-balance liability includes several time variables: time to maturity of the liability, duration of the assets underlying the crediting rate, and time between resets of the crediting rate. One might hope that one of these times would be the effective duration of the liability, but Tables 3 and 4 reveal a more subtle pattern. Examining why different likely candidates are not the correct duration of the liability is a good way to develop intuition for how the effective duration is determined.

The effective duration is not the time to maturity of the cash-balance liability. If a fixed amount were to be paid at the end, time to maturity would be the duration, but the amount paid is indexed to interest rates, and this indexing offers some protection against interest rate risk. That is, the claim is less sensitive to interest rate risk than a fixed claim at the same date, which implies a shorter effective duration.

The effective duration is positive, at least so long as the duration of the crediting asset is longer than the time between resets. Although one might conclude that the indexing provides full insurance against interest rate movements (implying a duration of zero), longer rates move less than the short rate; therefore, insurance is partial and the effective duration is typically positive.

The effective duration is not (except by accident) equal to the duration of the crediting asset. Crediting using a discount rate (a T-bill's discount to face value as a proportion of face value, divided by the fraction of a fictional 360-day year to maturity) can have some peculiar properties because the discounts themselves move less than the corresponding yields. However, crediting using yields always implies an effective duration less than the duration of the crediting asset. It might be tempting to think of the cash balance itself as being the valuation of a constant-maturity portfolio invested in the asset underlying the crediting rate. However, the cash balance is less sensitive to interest rates than the underlying constant-maturity portfolio, since it is not subject to the underlying portfolio's capital gains and losses.

The effective duration is not (except by accident) equal to the time between rate resets. Because duration in our analysis is always measured at a point in time just before a rate reset, one effect we have ignored is the change of duration between rate resets. When the time between resets is equal to the duration of the crediting asset and is also equal to the compounding interval, the effective duration is zero. In this special case, the duration between resets is the time until the next reset. Except in this special case, there is no reason that duration will equal the time until the next reset.

Hopefully, these observations suggest the subtleties of the determination of the effective duration. For crediting rates based on yields, the effective duration increases with the maturity of the liability and the maturity of the crediting asset. For all of these cases, the effective duration is less than either the maturity of the liability or the maturity of the crediting asset. For crediting based on Treasury discounts, durations can be larger (because discounts move less than yields) but the results are otherwise similar. In general, the duration changes little when the IRS margin is added.

Slope of the Yield Curve. Although not evident from the tables we have provided (because they are all based on the same yield curve), the slope of the yield curve does affect the cost of the liability. As illustrated in the example in the first section, crediting at a long-maturity rate in the presence of an upward-sloping yield curve implies a cost greater than the cash balance. In the yield curve of November 15, 1999, the implied forward rates rise until about 10 years out and then fall to near the starting value about 30 years out—which is why, of the liabilities without any margins, the 10-year liabilities have the highest cost and the 30-year liabilities have the lowest.

Uncertainty. One pleasant surprise is that the results in Tables 3 and 4 generally do not differ by much, so modeling the uncertainty is not very important. In terms of the theoretical literature on the term structure, the reason is that the convexity is not very important due to cancellation from similar movements in the crediting rate and the discount rate. The cancellation was evident in the example in the first section: As interest rates rise, the increase in the discount rates is offset by an increase in the crediting rates.[12]

Other Factors. We mentioned that the yield curve has a significant impact on cost; for this reason alone, the results in Tables 3 and 4 clearly cannot be applied directly for liabilities at any date other than November 15, 1999. Some of the choices in the simulation—for example, computing bond yields by using semi-annual compounding or computing Treasury discounts by using the customary formula based on a fictitious 360-day year—are based on industry conventions. Other choices are more arbitrary because there is no clear convention used in practice. And, changing these implementation details can affect the results. For example, using the average of rates over the last year instead of the quoted rate at the start of the year (as is used in the simulations) might change the cost and duration, especially when using a Treasury discount for the underlying rate. The choice of compounding interval for the liability will also affect the cost of the liability. The tables here are based on an intermediate case of quarterly compounding. If the liability compounded annually, the cost would be lower, whereas if the liability compounded continuously, the cost would be higher. Finally, recall that, by definition, the cost excludes the impact of early departures from the plan and any future contributions.

Cost of Cash-Balance Liabilities

Option-pricing theory provides a reliable approach to evaluating the cost of cash-balance liabilities. This approach gives formulas for discrete-time models or continuous-time models. Both types should give similar answers, and the choice of one type or the other seems to be a matter of convenience. Discrete-time models are handy for working with the available data (although fractional periods can be a nuisance); continuous-time models simplify compounding and tend to produce simpler exact formulas. We provide discrete-time formulas with occasional reference to continuous-time models.

Consider a liability with crediting rate r_t^c. Then, in the absence of any withdrawals or pay-related credits, the cash balance, B_t, satisfies

$$B_t = B_{t-1}(1 + r_t^c) \tag{5}$$

or, equivalently,

$$B_t = B_0(1 + r_1^c)(1 + r_2^c)(1 + r_3^c)\ldots(1 + r_t^c). \tag{6}$$

These equations assume that the crediting rate corresponds to the time interval for compounding. For example, if going from $t = 0$ to $t = 1$ is a year, r_t^c is an annual rate, and if going from $t = 0$ to $t = 1$ is half a year, then r_t^c is half the annual rate.

Modern financial theory says that, in the absence of arbitrage, cash flows are valued by using expected present values, computed using the realized spot rate for discounting and using artificial risk-neutral probabilities for computing expectations (see, for example, Dybvig and Ross 1987). If these probabilities are the actual probabilities, then the local expectations hypothesis holds and all assets have the same expected return. Otherwise, the risk-neutral probabilities are artificial probabilities that undo the risk premiums. For pricing interest derivatives, which assumption is made does not matter much so long as (1) the model for interest rates is consistent with the observed yield curve and (2) the volatility of interest rates is modeled well. Whichever the case, the expectation, $E(\bullet)$, is taken to be the appropriate expectation for valuation. Present values are computed by using the rolled-over spot rate, and therefore, the cost, C_0, of the liability with initial balance B_0 and horizon T is

$$\begin{aligned} C_0 &= E\left[\frac{B_T}{(1+r_1)(1+r_2)(1+r_3)\dots(1+r_T)}\right] \\ &= E\left[B_0\frac{(1+r_1^c)(1+r_2^c)(1+r_3^c)\dots(1+r_T^c)}{(1+r_1)(1+r_2)(1+r_3)\dots(1+r_T)}\right], \end{aligned} \tag{7}$$

where r_t is the spot interest rate quoted at $t - 1$ for payment at t (but is not the corresponding forward rate, which is stochastic).

Equation 7 shows that the cost equals the initial cash balance if the crediting rate equals the spot rate, $r_t = r_t^c$. In this case, each term in the numerator cancels the corresponding term in the denominator; therefore, $C_0 = E(B_0) = B_0$. A slightly more subtle observation is that if the crediting rate is a zero-coupon bond yield for a bond maturing at the next reset, the cost equals the initial cash balance in this case as well. The reason is that the zero-coupon bond yield, d_t^{t+M}, quoted at time t for maturity $t + M$ satisfies

$$\frac{1}{\left(1+d_t^{t+M}\right)^M} = E_t\left[\frac{1}{(1+r_{t+1})(1+r_{t+2})\dots(1+r_M)}\right] \tag{8}$$

or

$$1 = E_t\left[\frac{\left(1+d_t^{t+M}\right)\left(1+d_t^{t+M}\right)\dots\left(1+d_t^{t+M}\right)}{(1+r_{t+1})(1+r_{t+2})\dots(1+r_M)}\right], \tag{9}$$

where $E_t(\bullet)$ indicates expectation conditional on information at time t. Crediting at the zero-coupon bond yield over the period from t to $t + M$ means ratios of corresponding terms in the numerator and denominator of Equation 7 will have expectation 1, and from this observation it is not too difficult (using the law of iterated expectations) to complete a formal proof that crediting at the zero-coupon bond yield corresponding to the time between resets gives cost equal to cash balance.

To use Equation 7 when interest rates are random, we simulate both the spot rate and the crediting rate using a Vasicek term-structure model with means adjusted to fit today's yield curve, as described by Heath, Jarrow, and Morton (1992) or more explicitly by Dybvig (1997). Then, the appropriate coupon bond yield can be derived from the simulated forward rate curve using one of the formulas given in Table 1 of Dybvig, Ingersoll, and Ross (1996).

An alternative to using random interest rates is to assume that interest rates are not random and that the actual future interest rates are the forward rates implicit in today's Treasury strip curve. This assumption is obviously not literally true, but it simplifies the problem considerably. Although a little finesse is required to interpolate and extrapolate for maturities not observed and to smooth values made ragged because of bid–ask effects or asynchronous quotes, the basic idea is as follows. Take as given observations at time zero of the zero-coupon bond prices, D_0^t, for all t. Then, the implied forward rates, f_0^t, are given by the standard formula,

$$D_0^t = D_0^{t-1}\frac{1}{1+f_0^t} \tag{10}$$

or

$$f_0^t = \frac{D_0^{t-1}}{D_0^t} - 1, \tag{11}$$

because the absence of arbitrage implies the price is the same for a cash flow at time t whether it is bought directly (which has price D_0^t) or indirectly by buying cash at time $t - 1$ (at price D_0^{t-1}) to invest at the forward rate until t [at a price of $1/(1 + f_0^t)$ at time $t - 1$ for each dollar of cash at time t]. Assuming interest rates are nonrandom, then $r_t = f_0^t = f_s^t$ for all $s < t$; that is, borrowing or lending from $t - 1$ to t is the same whether contracted at $t - 1$ (at rate r_t), at zero (at rate f_0^t), or at s (at rate f_s^t). Otherwise, borrowing or lending forward with the offsetting trade in the spot market would be an arbitrage. Thus, the forward-rate curve has all the information needed to compute both the terminal balance, B_T, and the cost, C_0.

A Nifty Rule of Thumb. A useful rule-of-thumb generalizes the example given in the first section: Take as given annual forward rates f_0^1, f_0^2, . . ., computed from today's Treasury strip curve; assume the crediting rate is a three-year zero-coupon bond yield; and assume that maturity T is much larger than three years. Now, noting that compounding over a few periods is not very important, we invoke the approximation $(1 + x)(1 + y) \approx 1 + x + y$ for small x and y. Therefore, the crediting rate at t is

$$\begin{aligned} r_t^c &= \left[(1+f_0^t)(1+f_0^{t+1})(1+f_0^{t+2})\right]^{1/3} - 1 \\ &\approx \frac{f_0^t + f_0^{t+1} + f_0^{t+2}}{3} \end{aligned} \quad (12)$$

and, consequently,

$$\begin{aligned} C_0 &= B_0 \frac{(1+r_1^c)(1+r_2^c)(1+r_3^c)\ldots(1+r_T^c)}{(1+r_1)(1+r_2)(1+r_3)\ldots(1+r_T)} \\ &\approx B_0 \frac{(1+f_0^1/3)(1+2f_0^2/3)(1+f_0^3)\ldots(1+f_0^T)(1+2f_0^{T+1}/3)(1+f_0^{T+2}/3)}{(1+r_1)(1+r_2)(1+r_3)\ldots(1+r_T)} \\ &= B_0 \frac{(1+2f_0^{T+1}/3)(1+f_0^T/3)}{(1+2f_0^1/3)(1+f_0^2/3)} \\ &\approx B_0(1+f_0^T - f_0^1), \end{aligned} \quad (13)$$

where most of the approximations ignore compounding, and the last equality also assumes that the yield curve is reasonably flat at the beginning and the end. Intuitively, we are credited with all the rates in the middle, although partly displaced by a period or two. Therefore, all the middle rates cancel with the interest rates in the denominator, and the result is based on the difference between rates at the beginning and end.

More generally, a cash-balance liability credited at an M-period discount yield adjusted every period with a maturity $T > M$ has a cost of approximately

$$C_0 = B_0\left[1 + \frac{M-1}{2}(f_0^T - f_0^1)\right], \quad (14)$$

where $(M - 1)/2$ can be interpreted as half the mismatch of maturities between reset and crediting and $f_0^T - f_0^1$ can be interpreted as the slope of the (forward-rate) yield curve. Although not appropriate for critical applications requiring an exact number, this simple rule works surprisingly well.

Rule-of-thumb Equation 14 was derived under the assumption of no margin (IRS-prescribed or otherwise). Given a margin of π per unit time, the liability is larger by the factor of about $(1 + \pi)^T$ and Equation 14 becomes

$$C_0 = B_0\left[1 + \frac{M-1}{2}(f_0^T - f_0^1)\right](1+\pi)^T. \quad (15)$$

Effective Duration of Cash-Balance Plans

To compute the interest rate exposure of any fixed-income security or derivative, there must be some view, implicit or explicit, of how interest rate shocks today affect future rates. Our analysis uses the simple Vasicek model with mean reversion (with or without adjustments to fit the initial yield curve). Although this model is specialized and too simplistic for some purposes, it serves well in this context. In this model, a shock to today's interest rate is reflected linearly in future realized interest rates, with a declining effect over time. Specifically, a shock, δ, to the spot rate at time zero corresponds to an impact $\delta\exp(-kt)$ on the spot rate at t. The parameter k is the rate of mean reversion in interest rates; the larger k is, the more temporary the impact of the shock is and the smaller $\delta\exp(-kt)$ will be for each t. The effective duration is the maturity of the zero-coupon bond with both the same cost and the same sensitivity to δ as the claim.

One feature of the Vasicek model that simplifies the task significantly is the following formula for the impact of shock δ in the instantaneous interest rate on the zero-coupon bond price, D_0^t:

$$\frac{dD_0^t}{d\delta} = -\frac{1-e^{-kt}}{k}D_0^t; \quad (16)$$

therefore, solving for t produces

$$t = -\frac{1}{k}\log\left(1 + k\frac{1}{D_0^t}\frac{dD_0^t}{d\delta}\right). \quad (17)$$

Equation 17 expresses the effective duration of a zero-coupon bond in terms of its sensitivity to shock δ per unit of value. By the definition of effective duration (risk exposure per dollar of value is the same as for a zero-coupon bond with maturity at the claim's duration), the expression must be the same for all claims with duration t; therefore,

$$\begin{aligned} t &= -\frac{1}{k}\log\left(1 + k\frac{1}{C_0}\frac{dC_0}{d\delta}\right) \\ &= -\frac{1}{k}\log\left[1 + k\frac{d\log(C_0)}{d\delta}\right] \end{aligned} \quad (18)$$

is the effective duration of any liability whose cost is C_0. In the limit, as k tends to zero, no mean reversion occurs and Equation 18 becomes the same as what would be implied by the traditional Macauley duration based on parallel shifts of the yield curve.

There are a number of ways to compute the derivative in the effective duration formula (Equation 18). One simple way is to compute it numerically by considering directly shocks to the initial yield curve: Let D_0^t be implied from the initial given yield curve (perhaps a smoothed and extrapolated version of data on Treasury strips). Then, picking some small number for δ_0, consider the impact of positive and negative shocks of size δ_0 on the initial yield curve. Let

$$D_{0,up}^t \equiv D_0^t + \delta_0\left(-\frac{1-e^{-kt}}{k}D_0^t\right) \tag{19}$$

and

$$D_{0,down}^t \equiv D_0^t - \delta_0\left(-\frac{1-e^{-kt}}{k}D_0^t\right). \tag{20}$$

Then, Equation 16 and the definition of a derivative implies that

$$\frac{d\log(dC_0)}{d\delta} \approx \frac{C_{0,up} - C_{0,down}}{2\delta}, \tag{21}$$

where $C_{0,up}$ is based on $D_{0,up}$ and $C_{0,down}$ is based on $D_{0,down}$. Costs $C_0(D_{0,up})$ and $C_0(D_{0,down})$ can be computed numerically; then, substituting Equation 21 into Equation 18 gives the effective duration.[13] We used this approach to compute the effective duration numbers in Table 3.

Conclusion

This analysis of the market-value cost and effective duration of various cash-balance pension liabilities has considered term-structure effects and a variety of crediting rules that used market rates with or without margins given by IRS guidelines. The general theory explored here can be adapted to match specific institutional details that may differ from the assumptions we made or to include a model that incorporates random departures of beneficiaries.

A possible avenue for future study is to extend the analysis in this article to include employer and employee options to cash out the pension liability (at the cash value) by terminating employment. At a time when the economic value of pensions exceeds the cash value, companies may be quicker to terminate employees because of the consequent savings on pension expenses. Conversely, when cash value exceeds economic value, employees may have some incentive to change jobs to lock in the cash value. Employer and employee options may distort incentives but are not a direct threat to the pension insurance fund so long as the cash value is fully funded. In general, we do not believe these options are any more important for cash-value plans than for other defined-contribution plans, but this belief has yet to be proven.

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Notes

1. Payment of the cash balance on leaving the company, when the funds might be rolled into an IRA or another employer's plan, is subject to a vesting requirement for some companies, but usually over a relatively short period, no more than five years, as is typical in defined-contribution plans.
2. Actuarial assumptions could be used to combine these idealized units to mimic the payments of a whole plan. We are not dealing with the impact of employer and employee options to terminate employment. As we discuss in the conclusion, we do not expect these options to be any more important than for other types of defined-benefit plans, but this expectation is yet to be proven.
3. This issue is becoming more and more interesting because even T-bonds are becoming less liquid, in part as a result of the government's buyback program.
4. The PBO calculation for traditional defined-benefit plans differs from the market-value cost because the corporate bond rate is probably too high a discount rate for a liability collateralized by the assets in a plan. However, the effective durations and most cost comparisons of various crediting options are not much affected by this difference. The usual PBO calculation for a cash-balance plan is more convoluted because the final obligation does not depend on final salary, except indirectly through future pay-related credits.
5. The analysis does not vary the timing of resets from its customary value of once a year or the details of compounding within a year, both of which should be of minor importance.
6. IRS Notice 96-8, Internal Revenue Bulletin (1996-6):23–26.
7. These expressions are only approximate because compounding is ignored. The overall approximation is actually more accurate than it might seem because missing compounding in discounting and accumulation tends to cancel.
8. Consistent with the idea of a decreasing impact of a shock, we observe that the interest rate volatilities implicit in bond option contracts tend to decrease with the maturity of the underlying bond.
9. An objective comparison of the CPI case with the others is difficult because a primary determinant in the comparison would be beliefs about the future connection between interest rates and inflation.

10. Smoothing removes economically meaningless jumping around of forward rates that is likely the result of mispricing within the bid–ask spread. Extrapolation beyond the last strip is required for rates beyond the end of the current strip curve. If a program has 20 years to maturity and credits at a 30-year rate, computing the cost requires implied forward rates out 50 years. The smoothing is based on a least-squares fit of a quintic (fifth-order polynomial) to the implied forward-rate curve, constrained to be flat at the end. Rates are extrapolated smoothly using a flat forward-rate curve beyond the longest maturity of available strip prices.
11. See Dybvig (1997) for more on the Vasicek model with "fudge factors" to fit a given yield curve and also for estimates of volatility and mean reversion.
12. Convexity can matter a lot in valuation of long-maturity fixed-income obligations, as emphasized by Dybvig and Marshall (1996).
13. If this computation uses a simulation, the same random draws should be used for the up as for the down cases; otherwise, convergence will be slow.

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