Signalling by Possibly Incompetent Forecasters

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Abstract

Accepting a contract with a high incentive for performance is normally interpreted as a signal of high ability. However, an extremely high self-assessment may be an incompetent forecast by an incompetent worker. The authors do not want an investment advisor who expects nearly riskfree returns of 50%/year! We study separating equilibria in a model in which optimistic forecasters have low ability. In one type of equilibrium, a low incentive screens out the incompetent agents. In another, agents are wealthy enough for the principal to select the incompetent agents who cover the downside (as in a vanity press).

1 Introduction

Models in which agents signal ability typically assume that beliefs come from common priors.[1] In these models, incentive pay tends to attract agents of higher ability and screens out weaker candidates. For example, in the context of investment portfolio management, Bhattacharya and Pfleiderer (1985) show that, given common priors about managers' abilities, more competent managers self-select into positions with higher pay-for-performance. In practice, however, there are no contests selecting portfolio managers who signal they are best by taking higher risk. We show that allowing agents to have different priors about their abilities permits a simple and plausible explanation for why this does not happen. Agents who are relatively pessimistic about their ability are likely to avoid jobs with any incentive pay, while agents who are very optimistic will self-select themselves for jobs with big incentive pay. This may be a problem for employers expecting to attract good workers, especially when documented ability is scarce and self-assessed ability is great (as in the portfolio management industry). In this environment, instead of searching for competent managers, employers may prefer to hire overly optimistic agents, but only if these agents have significant assets to pledge. An agent with a strong belief in own ability will be willing to guarantee the performance with personal wealth, and it will not matter if the employer thinks the agent cannot deliver. A good example of this is a vanity press that charges authors to publish their work.[2] When such guarantees are not feasible (because the potential employee is not rich enough, the typical situation in money management),

[1]See, for example, Spence (1973), Bhattacharya and Pfleiderer (1985), Koszegi, Botond, and Li (2008).

[2]Some authors who use a vanity press do not care about sales and simply want to produce a beautiful book, possibly just to give their friends. These are not the people in our models.

employers may have to offer inefficiently low performance incentives to screen out overly optimistic employees.

Our main model has a principal who seeks to hire an agent to fill a single position. We study separating equilibria in a pure screening setting (hidden exogenous type but no hidden effort) with two types of potential manager.[3] From the perspective of the principal, one type is a good manager and is only slightly over-confident, while the other type is not a good manager but is extremely over-confident. Except in welfare discussions, it does not matter which agent, if any, has "correct" beliefs, but it is perhaps useful to take the view of the principal that all agents are overly optimistic and the more optimistic type of potential manager is incompetent. We show that, to attract the more competent agents, the principal offers a wage schedule with little incentive compensation, making it unattractive to the incompetent but extremely confident agents. The principal, however, finds attracting incompetent agents more profitable if the agents have large pledgeable wealth (implying that the lower bound on wage is negative and large in absolute value). In this case, the principal offers a contract with a big embedded bet on output. Such a bet is attractive only to the extremely confident, incompetent type. We show that our results are robust to adding pessimistic and incompetent agents, and extend to the setting with multiple positions.

When perfect screening of agents is not possible, the remaining uncertainty about the manager's overconfidence and ability will affect how much the principal can learn from the manager's forecasts. We study the issue by abstracting from adverse selection and as-

[3] In addition to separating equilibria, the model can in general also have pooling equilibria in which either everybody or nobody applies for the position, but the pooling equilibria are less interesting to us.

suming that the agent issues honest reports. We show that extreme forecasts are rationally interpreted by the principal as indicative of overconfidence rather than true future performance. For some parameter values, an extremely positive forecast can be interpreted by the principal as indicating worse future performance than a moderately positive forecast.

It is in principle possible to test our model empirically by studying the link between incentives in compensation and portfolio performance. As for most information models, however, it is difficult to construct a robust test with the available data. For example, while some studies find that stronger monetary incentives correspond to better performance (for example, Khorana, Servas, and Wedge (2007)), it is hard to tell from the evidence whether the positive relationship is due to effort or ability, and it is especially difficult to link the results to the managers' perceptions of their ability. A number of empirical measures of managerial overconfidence are offered in Malmendier and Tate (2005); evidence of managerial overconfidence is also reported in Ben-David, Graham, and Harvey (2007). They, however, do not look at the link between their measures and CEO compensation packages, and that may be one place to look for evidence that can be used to test our model.

Our paper is part of the growing literature which studies agents who have different prior beliefs. Santos-Pinto and Sobel (2005) illustrate where differences in beliefs might come from. Van den Steen (2004) discusses consistency between individual rationality and differences in beliefs, and shows that agents with different priors tend to overestimate their ability to control the outcome, achieve success, and outperform others. Van den Steen (2005) focuses on team structure and argues that agents with similar priors are more likely to self-select into the same firm. The effect of managerial overconfidence (which can be viewed as disagreement with the principal) on the firm investment policy and manager's welfare is studied in Gervais, Heaton, and Odean (2007). Adrian and Westerfield (2008) use a dynamic setting to analyze how disagreement between the agent and the principal impacts the optimal risk sharing. The possibility that introducing agents with biased ability estimates may significantly affect the optimal compensation structure in a signalling model is noted by Dybvig, Farnsworth, and Carpenter (2004), but they do not develop this point. We contribute to the literature by studying self-selection in the presence of fundamental disagreement about agents' abilities.

This paper is organized as follows. Section 2 describes the model and derives the equilibria. The principal's interpretation of forecasts of an overconfident manager are analyzed in section 3. Section 4 shows that our model results are robust to various extensions, and section 5 concludes. The formal proofs are in the Appendix.

2 Model

There is a risk-neutral principal who wishes to hire one manager. There are two types of agents who can become managers: type o (optimistic, overconfident) and type c (conservative, competent). The type of each agent is known to the agent but not to the principal. Let N be the finite total number of agents, let π_o be the proportion of agents that are type o, and let $\pi_c = 1 - \pi_o$ be the proportion of agents that are type c. All agents are risk-neutral over nonnegative wealth. The firm's output equals the manager's ability $a \in \{a_l, a_h\}$, where $a_h > a_l$. The agents' true ability is unknown to both the principal and the agents. Type o (resp. c) agents believe that their ability is high with probability q_o (resp. q_c), while the principal believes that their ability is high with probability f_o (resp.

f_c). Beliefs about probabilities for various types are public knowledge,[4] (although agent types are private information) and we make no assumptions about the true distribution of the agents' abilities. For example, the equilibrium is the same whether type o agents have unrealistically optimistic expectations or the principal just doesn't appreciate how good they are. When we discuss the model results, however, we often take the view of the principal, as it seems natural and helps the exposition.

We assume that $f_o < f_c < q_c < q_o$, which implies that both types of agents place a larger probability on their ability to be high than the principal does, and the principal believes that overly confident agents are less likely to have high ability. This is the interesting case, although the intuition is similar when the agents are less optimistic than the principal. As we discuss in section 4, adding a pessimistic ($q_p < q_c$) and incompetent ($f_p < f_o$) agent with a reasonably high reservation utility would not change the equilibrium because such an agent would not accept the optimal contracts we describe.

The principal advertises the managerial position with the wage schedule $w = (w_l, w_h)$, where compensation is w_l if the output is a_l and w_h if the output is a_h. We require the wage to be nondecreasing in the output: $w_h \geq w_l$ (a decreasing wage would give agents an incentive to destroy part of the output to receive a higher wage). All agents choose simultaneously whether to apply. We denote the decision of a type o (resp. c) agent to apply for wage schedule w using function $m_o(w)$ (resp. $m_c(w)$) which takes on the value of 1 if the agent applies and 0 if the agent does not. Note that functions $m_o(w)$ and $m_c(w)$ are defined for any feasible wage w (in an extensive form game, agents specify responses to both on-equilibrium and off-equilibrium actions of other agents). If only one

[4]In the model, the candidate managers are price-takers and thus do not need to know other agents' abilities and beliefs.

agent applies, the agent is hired. If several agents apply, the principal randomly picks an agent. If no agent applies, no one is hired and the principal receives a normalized zero payoff. We employ the solution concept of *subgame perfect Bayesian Nash equilibrium* in symmetric pure strategies, where by symmetric we mean agents of the same type play the same strategy. In this game, the only proper subgames are the whole game and the subgames starting after the wage schedule is locked in, when the agents choose whether to apply.

We assume that each agent is endowed with the same initial wealth $W \geq 0$ (because we are not studying screening on initial wealth), so we impose the following limited resources constraint: $w_h, w_l \geq -W$. All type o (resp. c) agents not hired by the principal receive expected wage u_o (resp. u_c) in the outside market. Therefore, given other agents' application strategies, the agent of type $j \in \{o, c\}$ maximizes expectation of utility equal to

$$(1) \quad \begin{cases} q_j w_h + (1 - q_j) w_l & \text{if hired;} \\ u_j & \text{if not hired.} \end{cases}$$

Given there are finitely many agents, the probability of getting hired is 0 if the agent does not apply ($m_j(w) = 0$) and positive if the agent applies ($m_j(w) = 1$), whatever the strategies played by the other agents. Therefore we do not need to do any detailed calculations to show that a strategy is optimal if and only if it satisfies

$$(2) \quad m_j(w) \begin{cases} = 1 & \text{if } q_j w_h + (1 - q_j) w_l > u_j; \\ \in [0, 1] & \text{if } q_j w_h + (1 - q_j) w_l = u_j; \\ = 0 & \text{if } q_j w_h + (1 - q_j) w_l < u_j, \end{cases}$$

for all w. Note that, since we focus on equilibrium in pure strategies, we can restrict attention without loss of generality to $m_j(w) \in \{0, 1\}$. The payoff for the principal from offering w is

$$(3)\quad \begin{cases} 0 & \text{if } m_o(w) = m_c(w) = 0 \\ f_o(a_h - w_h) + (1 - f_o)(a_l - w_l) & \text{if } m_o(w) = 1 \text{ and } m_c(w) = 0; \\ f_c(a_h - w_h) + (1 - f_c)(a_l - w_l) & \text{if } m_o(w) = 0 \text{ and } m_c(w) = 1; \\ \sum_{j=o,c} \pi_j (f_j(a_h - w_h) + (1 - f_j)(a_l - w_l)) & \text{if } m_o(w) = m_c(w) = 1. \end{cases}$$

The following restrictions on the parameter space let us focus on cases that make our economic point.

Assumption 1. *a. The principal believes that all agents are less skilled than they think, and that more optimistic agents (type o) are less skilled than more conservative agents (type c):*

$$f_o < f_c < q_c < q_o.$$

b. Even a low-ability manager produces enough output to make the firm profitable:

$$a_l > u_o.$$

c. Reservation utility of type o is above that of type c:

$$u_o > u_c.$$

d. Reservation utility of type o is not too large:

$$\frac{u_o + W}{q_o} \leq \frac{u_c + W}{q_c}.$$

Assumption 1b is sufficient to ensure that the principal will hire someone in equilibrium. As we show in the proof of Theorem 1, assumption 1d, which imposes an upper boundary on the reservation utility u_o, is a necessary condition for hiring type o in equilibrium. If u_o were too large, the model would become trivial: the principal would only hire the less confident type c because this type would be both cheaper and more productive.

2.1 Equilibrium

Given our parameter restrictions specified in Assumption 1, there are two possible types of symmetric separating equilibria.[5] The following theorem characterizes these equilibria.

Theorem 1. *Given Assumption 1, there are two possible types of separating equilibrium, Equilibrium VP and Equilibrium TS, defined as follows.*

Equilibrium VP (Vanity Press)*: only the less competent (according to the principal) type o agents apply. In this equilibrium, the principal offers wage* $w^* = (w_l^*, w_h^*)$ *with*

$$w_h^* = \frac{u_o + W(1 - q_o)}{q_o}, \qquad w_l^* = -W. \tag{4}$$

The agents' responses are given by (2) with $m_c(w^*) = 0$, $m_o(w^*) = 1$. *The principal hires a type o agent and receives the following payoff:*

$$\Pi_{vp} = f_o a_h + (1 - f_o) a_l - \frac{f_o}{q_o} u_o + \frac{q_o - f_o}{q_o} W; \tag{5}$$

Equilibrium TS (Talent Screening). *Only the more competent (according to the principal) type c agents apply. In this equilibrium, the principal offers wage* $w^* = (w_l^*, w_h^*)$

[5] For some parameter values, there may exist a pooling equilibrium where all agents apply, and the principal hires an agent of type j with probability π_j. We view this case as less interesting because it is qualitatively similar to what we can have in a traditional model with common beliefs. Nevertheless, we offer the formal specification and the conditions for the existence of a pooling equilibrium in the Appendix. Additionally, absent Assumption 1, there can be a pooling equilibrium in which no agent applies for the position, for example if both have reservation wages that are too high.

with

$$(6)\quad \begin{aligned} w_h^* &= \frac{(1-q_c)u_o - (1-q_o)u_c}{q_o - q_c}, \\ w_l^* &= \frac{q_o u_c - q_c u_o}{q_o - q_c}. \end{aligned}$$

The agents' responses are given by (2) with $m_c(w^*) = 1$, $m_o(w^*) = 0$. The principal hires a type c agent and receives the following payoff:

$$(7)\quad \Pi_{ts} = f_c a_h + (1-f_c)a_l - \frac{(q_o - f_c)u_c - (q_c - f_c)u_o}{q_o - q_c}.$$

Proof: See the Appendix.

Because the problem faced by the principal is linear, the wages that appear in the equilibria are at extreme points. We put enough structure on the model to make sure that both types of separating equilibrium can occur, as illustrated in Figure 1. In this figure, the two solid lines represent the agents' participation constraints. The axes are drawn to start at $(-W, -W)$, and therefore, each line represents the wealth the agent will have at the end including the initial endowment of W. Because the optimistic agent's reservation utility is assumed to be above the more conservative agent's reservation utility, the lines cross above the 45 degree line. The dashed lines in the figure represent the principal's indifference curves. Note that the principal's indifference curves are steeper than those of either agent because the principal in our model has the most conservative beliefs about the agents' abilities. Moreover, the slope of the principal's indifference curve depends on which agent the principal expects to hire. The indifference curve corresponding to hiring type o is the steepest, since the principal is the most skeptical about the ability of type o.

The principal wishes to minimize the expected wage expenditure. The smallest expected compensation package that attracts type o is represented in the figure by the point called

"Equilibrium VP". This package requires type o to post wealth W as a guarantee of high outcome. The agent has positive wealth only if the outcome is indeed high. This positive wealth level is just sufficient to provide the agent with reservation utility in expectation. Similarly, the smallest expected compensation package that attracts agent c is represented in the figure by the point called "CO" (stands for "competent only"; this compensation would be used if all agents in the market were competent). This compensation package, however, also attracts type o (it is above type o's reservation utility line).

For the principal, the cheapest way to attract type c without attracting type o is to offer the compensation package denoted in the figure as "Equilibrium TS". Technically, given the wage in "Equilibrium TS", type o is indifferent between applying and not applying. However, it is not an equilibrium for type o to apply: if both agents apply, the principal wishes to deviate and offer a wage that lies above type c's reservation line but below type o's reservation line. This wage is only marginally more expensive to the principal, and is guaranteed to attract only type c. Note that this wage has low performance sensitivity; in the extreme case when the agents have the same reservation utility: $u_o = u_c$, the equilibrium wage is flat: $w_h = w_l = u_c$. This result contrasts that of Bhattacharya and Pfleiderer who suggest that more able managers are attracted by higher performance sensitivity: given that better managers are more measured forecasters in our model, they are attracted by small performance sensitivity. The expected level of the equilibrium wage attracting type c is decreasing in type o's reservation utility and increasing in type o's confidence q_o. Thus, hiring the type c is more expensive (in absolute terms, not relative to hiring other agents) when the agent's type o competitors are more optimistic or are willing to work for less.

Comparing the principal's payoffs Π_{vp} and Π_{ts} given by (5) and (7) respectively suggests that the principal chooses to hire type o (Equilibrium VP occurs) when agents have high initial wealth W. Intuitively, because of type o's lack of ability ($f_o < f_c$) and the higher reservation utility $u_o > u_c$, hiring a type o agent is desirable only when the value of arbitraging the difference in beliefs is high, which is true when W is large. Being overly optimistic, type o is willing to bet all the cash on high outcome. The other side of this bet is attractive for the principal who believes high outcome is not very likely. For example, suppose that $W = 1$ and let us also assume that $\pi_o = 0.9$, $u_o = 2.1$, $u_c = 2$, $q_o = 0.6$, $q_c = 0.5$, $f_o = 0.1$, $f_c = 0.3$, and $a_h - a_l = 10$. Substituting these parameters into (5) and (7) produces $\Pi_{vp} > \Pi_{ts}$, implying that Equilibrium VP is the unique separating equilibrium, where only the optimistic type o is hired. If, however, we reduce the agents' initial wealth to $W = 0.5$, the assumed parameters lead to $\Pi_{ts} > \Pi_{vp}$, and Equilibrium TS becomes the unique separating equilibrium, where only the competent type c is hired.[6] Comparing (5) and (7) also suggests that the principal may choose to hire type o even when agents have no initial wealth ($W = 0$) but type o is extremely optimistic ($q_o \gg f_o$). In this case, the principal views type o as cheap: the required compensation is positive only in the unlikely event of high outcome.

Theorem 1 is proven under Assumption 1 that in particular requires the reservation utility of the optimistic type o to be sufficiently low (assumption 1d). If this assumption were violated, the solution would be trivial: the principal would find it optimal to hire only the conservative but competent type c by offering a wage that is positive only if the output is high and offers a reservation utility level to the competent agent. To see that, observe that if the inequality in 1d were violated, the participation constraint for type o would lie

[6] It can also be verified that there is no pooling equilibrium with the above parameter values for both $W = 0.5$ and $W = 1$.

fully above the participation constraint for type c (hence, unlike in Figure 1, in this case, the two participation constraints would not intersect). Thus, the principal would view type o as both more expensive and less productive.

3 Forecasts of Overconfident Managers

Outside the context of our specific model, the possibility that forecasters may be incompetent can color our interpretation of their forecasts. Suppose agents report honestly their best forecasts for some variable. If we know an agent's type, then we would interpret the agent's report differently depending on the type. If we don't know the agent's type, then we try to infer the type from the forecast. For example, if an agent forecasts the stock market will go up 50% tomorrow, then we will infer that this agent is probably an incompetent forecaster, and not somebody with extremely positive useful information. As a result, our own expectation given the 50% forecast may be below the expectation given a 5% forecast, which is more likely to be based on useful information.

Suppose that the manager's task is to report to the principal the expected value of a random variable $X \sim N(0, \sigma_x^2)$. The manager of type $j \in \{o, c\}$ receives a noisy signal $s = X + \varepsilon$ about the variable X, where $\varepsilon \sim N(0, \sigma_j^2)$. Note that, unlike in our model of section 2, both agents are forecasting the same variable, and not their own ability. After observing the signal s, the manager issues a truthful report R that reflects the manager's (possibly biased) belief about the expected value of X:

$$R = K_j E[X|s].$$

We assume that type o agents have a larger upward bias: $K_o > K_c \geq 1$ and receives a noisier signal: $\sigma_o < \sigma_c$. Note that more noise in the signal (larger σ_j) leads to a smaller variance of the report R. Thus, we additionally assume that K_o is sufficiently large so that the overly optimistic agent o issues more volatile reports than the more conservative agent c.

The report R is informative both about the manager's type j and about the variable of interest X. Assume that the agent is of type o with probability π_o. After observing report R, Bayes' law implies:

$$\Pr(\text{type } o|R) = \frac{\phi(R|0, \hat{K}_o^2\sigma_x^2 + \hat{\sigma}_o^2)\pi_o}{\phi(R|0, \hat{K}_o^2\sigma_x^2 + \hat{\sigma}_o^2)\pi_o + \phi(R|0, \hat{K}_c^2\sigma_x^2 + \hat{\sigma}_c^2)(1 - \pi_o)},$$

where $\hat{\sigma}_j = \sigma_j K_j \sigma_x^2/(\sigma_x^2 + \sigma_j^2)$, $\hat{K}_j = K_j\sigma_x^2/(\sigma_x^2 + \sigma_j^2)$, and $\phi(z|\mu, V)$ is the normal probability density function with mean μ and variance V calculated at z. Because type c has smaller signal variance than type o has, extreme reports R are indicative of a type o manager.

Given report R, conditional distribution of X is a mixture of two gaussian distributions: $X|R \sim N(\bar{X}_o, v_o^2)$ with probability π_o and $X|R \sim N(\bar{X}_c, v_c^2)$ with probability $\pi_c = 1 - \pi_o$, where

$$\bar{X}_j = \frac{\hat{K}_j\sigma_x^2}{\hat{K}_j^2\sigma_x^2 + \hat{\sigma}_j^2}R; \qquad v_j = \frac{\hat{\sigma}_j^2}{\hat{K}_j^2\sigma_x^2 + \hat{\sigma}_j^2}\sigma_x.$$

The expected level $E[X|R]$ is shown in Figure 3. The figure assumes that agent c does not overstate s on average ($K_c = 1$), but agent o does ($K_o = 10$). Because the identity of the agent is not known, we expect that, on average, R overstates the true absolute value of X ($|E[X|R]| < |R|$). Moreover, the expected overstatement is larger for larger values of R ($E[X|R]$ is flatter for larger $|R|$). This is because when $|R|$ is large, the agent is more likely to be of type o. The impact of the report on the posterior expected value of X also depends on the relative signal informativeness σ_o and bias K_o of the optimistic agent. In the limit, as type o signal becomes relatively less informative and the report becomes relatively more biased (σ_o and K_o increase while σ_c and K_c remain constant), both the slope and the level of $E[X|R]$ tend to zero for every R.

4 Extensions

In our model of section 2, the principal has only one position, there are two types of candidates, and there are many candidates of each type. In this section, we show that the results are robust to introducing additional types and multiple positions. Additionally, this section discusses the existence and the shape of the pooling equilibrium in our main model.

4.1 Adding Pessimistic and Incompetent Agents

Introducing additional type p agents who are pessimistic and incompetent, and have reasonably high reservation wage ($u_p \geq u_c$ is sufficient but not necessary), would not affect the equilibria described in Theorem 1. So long as the participation constraint for the new type of agent is above the two equilibria depicted in Figure 1, the new type of agent will not apply for any wage contract that appears in Equilibria VP and TS. Because the principal finds the new agent more expensive and less able that the other agents, the principal will not alter the wage contracts to attract the new agent. Adding other types of agents, however, may have some effect on the equilibrium. For example, adding a moderately confident agent with sufficiently low reservation utility will result in this agent accepting all wage contracts that can possibly attract anyone else (if the participation constraint line of the new agent is well below the equilibria depicted in Figure 1). If the principal views the new agent's ability as sufficiently low, the principal does not want to hire the agent, and mechanisms other than wage structure would be required to screen out these agents.

4.2 Scarce Agents, Many Positions

When the principal has many positions and there is a shortage of agents, the wages that arise in equilibrium are similar to those highlighted in Figure 1, as we show next.

Suppose that at each time $t > 0$, the principal is approached by one agent. The principal believes that the agent is type o with probability π_o and type c with probability $1 - \pi_o$ (thus, knowing the types of past applicants does not inform the principal about the type of the current applicant). The principal offers the agent a menu of wage schedules, and the agent then decides whether to apply for one of them. Given two agent types, we can assume without loss of generality that the menu of wages consists of at most two wage schedules: one to attract type o and one to attract type c.

If the principal wishes to hire every agent, the principal's objective is to find the least expensive pair of wage schedules that would attract both types. The principal's problem is similar to the principal's problem in section 2, and thus leads to essentially the same wage schedules (we omit the formal derivation because it closely resembles that for Theorem 1). Specifically, depending on parameter values, the principal finds it optimal to either offer one wage schedule represented by point "CO" in Figure 1 to every agent, or offer a pair of wage schedules represented by points "Equilibrium VP" and "Equilibrium TS" in Figure 1, and let each agent choose which wage schedule to apply for. Hiring only one agent type is not an equilibrium in this setting because the principal could benefit by switching to hiring both types using one of the above wage menus (Assumption 1 guarantees feasibility of both wage menus). Specifically, hiring only type o (with wage schedule "VP") is dominated by hiring both types with wage schedules "VP" and "TS," and hiring only type c (with wage schedule "TS") is dominated by hiring both types with wage schedule "CO". Thus, there are two types of pure strategy symmetric equilibria:

Equilibrium CO At each time t, the principal offers one wage given by

$$(8) \quad \begin{aligned} w_h^* &= \frac{u_c + W(1-q_c)}{q_c}, \\ w_l^* &= -W; \end{aligned}$$

the agent that arrives at time t applies and is hired with wage w^*.

Equilibrium TS_VP At each time t, the principal offers a menu of two wages, w_o^* and w_c^*, given by

$$(9) \quad \begin{aligned} w_{oh}^* &= \frac{u_o + W(1-q_o)}{q_o}, \\ w_{ol}^* &= -W \end{aligned}$$

and

$$(10) \quad \begin{aligned} w_{ch}^* &= \frac{(1-q_o)u_c - (1-q_c)u_o}{q_c - q_o}, \\ w_{cl}^* &= \frac{q_c u_o - q_o u_c}{q_c - q_o}. \end{aligned}$$

If the agent that arrives at time t is type o, the agents applies for wage w_o^* and is hired; if the agent is type c, the agent applies for wage w_c^* and is hired.

In Equilibrium CO, the wage is represented by point "CO" in Figure 1. Thus, in this equilibrium, the agent of type c receives a reservation wage while the optimistic agent receives wage above the reservation level. Both agents are compensated only if the output is high, implying high performance sensitivity. In Equilibrium TS_VP, the equilibrium wage is represented by points "Equilibrium VP" and "Equilibrium TS" in Figure 1. Thus, in this equilibrium, both agent types receive a reservation wage. The wage of agent c has low performance sensitivity: the agent is compensated both when output is high and and when it is low. The wage of agent o, on the other hand, has high performance sensitivity: the agent is compensated only when the output is high. Note that hiring all agents allows the principal to let at least one type of agent bet their wealth on the outcome. This is an attractive bet to the principal, whose beliefs are more conservative than those of the agents.

As in the main model, which equilibrium is realized depends on the model parameters. Because in both equilibria the principal hires all agents, the principal chooses the equilibrium with the lowest expected wage. From the principal's point of view, hiring type o is relatively cheaper, and hiring type c is relatively more expensive, in equilibrium TS_VP than in equilibrium CO. Thus, equilibrium TS_VP occurs when there are relatively many agents of type o (π_o is high), or when the differences in agents' beliefs are large (q_o is a lot larger than q_c).

4.3 Pooling Equilibrium

We are primarily interested in the separating equilibria in the model, but it is worth having a look at the pooling equilibrium. The following lemma states that any pooling equilibrium in the model will be found at the wage schedule CO in Figure 1. This wage is sensitive to performance and results in a positive rent to type o agents.

Lemma 1. *If there exists a pooling equilibrium NS ("no screening") where both agent types apply:* $m_c(w^*) = m_o(w^*) = 1$*, then the wage* $w^* = (w_l^*, w_h^*)$ *offered in this equilibrium is given by*

$$w_h^* = \frac{u_c + W(1 - q_c)}{q_c}, \qquad w_l^* = -W; \tag{11}$$

and the principal's payoff is:

$$\Pi_{ns} = f_m a_h + (1 - f_m)a_l - \frac{f_m}{q_c}u_c + \frac{q_c - f_m}{q_c}W \tag{12}$$

where $f_m \equiv f_o\pi_o + f_c\pi_c$ *is the probability (according to the principal) that the hired agent has high ability.*

Proof: See the Appendix.

Existence of the pooling equilibrium depends on the model parameters. Intuitively, the pooling equilibrium occurs when hiring type c is attractive to the principal (for example because f_c is large), and there too few type o agents (π_o is small) to justify screening. In general, any combination of the equilibria discussed in the context of our main model can occur for some parameter values, as shown in the following lemma.

Lemma 2. *Equilibria VP, TS, and NS defined in the statements of Theorem 1 and Lemma 1 can occur in any combination.*

Proof. Lemma 3 (in the Appendix) shows that the optimal way to attract type o is by offering wage schedule (4), and results in payoff (5), while the optimal way to attract type

c is by offering wage schedule (6), and results in payoff (7). From Lemma 1, the optimal way to attract both types is by offering wage schedule (11), which results in payoff (12) (by assumption 1, attracting no agents is not beneficial for the principal). Therefore, in equilibrium, the principal chooses out of these three wage schedules the wage schedule (or schedules) that results in the highest payoff. Depending on parameter values, any or all of the three sets of strategies can form an equilibrium, as we show using numerical examples summarized in Table 1. □

5 Conclusion

This paper looks at compensation mechanisms that attract talented managers when the principal disagrees with the agents' assessments of their own ability. In contrast with the common belief that stronger incentives attract more talented managers, we show that stronger incentives may instead attract overly confident managers. The principal may benefit from hiring the more confident instead of the more talented agent if the confident agent has deep pockets and is willing to bet the wealth as well as the future compensation on success.

Appendix

This Appendix gives proofs for the main model presented in Section 2.

Proof of Lemma 1.

Proof. Consider a pooling equilibrium that attracts both types of agents. The wage schedule in this equilibrium must be better for the principal than any other wage schedule. In particular, it must be better than any other wage schedule that attracts both types of agents. Therefore, the equilibrium wage schedule must solve the following problem:[7]

Problem 1.

$$\min_{w}(f_m w_h + (1-f_m)w_l) \qquad \textit{subject to}$$

$$
\begin{aligned}
(13)\quad u_o &\leq q_o w_h + (1-q_o)w_l,\\
(14)\quad u_c &\leq q_c w_h + (1-q_c)w_l,\\
(15)\quad w_l &\geq -W,\\
(16)\quad w_h &\geq w_l,
\end{aligned}
$$

where $f_m \equiv f_o\pi_o + f_c\pi_c$. In this problem, the constraints (13) and (14) insure that the best response of both types is to apply, as described in (2).

The solution $w^* = (w_l^*, w_h^*)$ to Problem 1 satisfies the following first order conditions with

[7] While the objective in Problem 1 assumes that both agent types apply, some agents will be indifferent between applying and not applying if either (13) or (14) is satisfied with equality. Thus, we have to show that there is no equilibrium that does not solve Problem 1, in which the agents who are indifferent about the wage schedule solving Problem 1 do not behave as planned. The wage schedule in such an equilibrium would cost more to the principal than the wage schedule $w^* = (w_h^*, w_l^*)$ that solves Problem 1, and therefore would be dominated by the wage schedule $\hat{w} = (w_h^*, w_l^* + \varepsilon)$ with small enough $\varepsilon > 0$, which satisfies both (13) and (14) with strict inequality, and costs less to the principal for small enough $\varepsilon > 0$.

respect to w_h and w_l and complementary slackness conditions:

$$\begin{aligned}
f_o &= \lambda_1 q_o - \lambda_2 q_c; \\
1 - f_o &= \lambda_1(1-q_o) - \lambda_2(1-q_c) + \lambda_3; \\
0 &= \lambda_1(q_o w_h + (1-q_o)w_l - u_o); \\
0 &= \lambda_2(u_c - q_c w_h - (1-q_c)w_l); \\
0 &= \lambda_3(W + w_l); \\
0 &= \lambda_4(w_h - w_l); \\
\lambda_k &\geq 0, \text{ k=1,2,3,4.}
\end{aligned}$$

Wage schedule w^* given by (11), combined with $\lambda_1 = \lambda_4 = 0$, $\lambda_2 = f_o/q_c$, and $\lambda_3 = 1 + f_o/q_c$, satisfies the above conditions. Therefore, w^* is the equilibrium wage schedule. Uniqueness of the solution follows from Lemma 4 (see the end of the Appendix). □

Lemma 3. *Every separating equilibrium is either of the form VP or TS defined in the statement of Theorem 1.*

Proof. By definition, a separating equilibrium must attract either all agents of type o and none of type c or all agents of type c and none of type o. We analyze each of these two kinds of equilibrium separately,[8] starting with the equilibrium that attracts only type o but not type c (the analysis of the other case is similar). The wage schedule in this equilibrium must be better for the principal than any other wage schedule. In particular, it must be better than any other wage schedule that attracts only agents of type o. Therefore, the

[8] In this lemma, we set aside the question of existence of such equilibrium. Existence is addressed in Lemma 2.

equilibrium wage must solve the following problem:[9]

Problem 2.

$$\min_{w}(f_o w_h + (1 - f_o)w_l) \qquad \textit{subject to}$$

$$
\begin{array}{llll}
(17) & u_o & \leq & q_o w_h + (1 - q_o)w_l, \\
(18) & u_c & \geq & q_c w_h + (1 - q_c)w_l, \\
(19) & w_l & \geq & -W, \\
(20) & w_h & \geq & w_l,
\end{array}
$$

where the constraints (17) and (18) insure that the best response of type o is to apply and the best response of type c is to not apply, as described in (2).

The solution $w^* = (w_l^*, w_h^*)$ to Problem 2 satisfies the following first order conditions with

[9] While the objective in Problem 2 assumes that only type o applies, some agents will be indifferent between applying and not applying if either (17) or (18) is satisfied with equality. This is not a problem, however, which can be seen following the logic of footnote 6.

respect to w_h and w_l and the complementary slackness conditions:

$$\begin{aligned}
f_o &= \lambda_1 q_o - \lambda_2 q_c;\\
1 - f_o &= \lambda_1(1-q_o) - \lambda_2(1-q_c) + \lambda_3;\\
0 &= \lambda_1(q_o w_h + (1-q_o)w_l - u_o);\\
0 &= \lambda_2(u_c - q_c w_h - (1-q_c)w_l);\\
0 &= \lambda_3(W + w_l);\\
0 &= \lambda_4(w_h - w_l);\\
\lambda_k &\geq 0,\ \text{k=1,2,3,4.}
\end{aligned}$$

Wage schedule (4) combined with $\lambda_2 = \lambda_4 = 0$, $\lambda_1 = f_o/q_o$, $\lambda_3 = 1 - f_o/q_o$ satisfies the above conditions, and is therefore the equilibrium wage schedule. Uniqueness of the solution follows from Lemma 4 (see the end of the Appendix).

Consider next the equilibrium that attracts only type c but not type o. Similarly to the previous case, the wage in this equilibrium must solve the following problem:[10]

Problem 3.

$$\min_w (f_c w_h + (1-f_c)w_l) \qquad \textit{subject to}$$

[10]While the objective in Problem 3 assumes that only type c applies, some agents will be indifferent between applying and not applying if either (21) or (22) is satisfied with equality. This is not a problem, however, as can be seen following the logic of footnote 6 and using $\hat{w} = (w_h - \varepsilon/q_c, w_l + \varepsilon/(1-q_c) + (q_o - q_c)\varepsilon/q_c)$.

$$
\begin{aligned}
(21)\quad u_o &\geq q_o w_h + (1-q_o)w_l, \\
(22)\quad u_c &\leq q_c w_h + (1-q_c)w_l, \\
(23)\quad w_l &\geq -W \\
(24)\quad w_h &\geq w_l,
\end{aligned}
$$

where the constraints (21) and (22) insure that the best response of type c is to apply and the best response of type o is to not apply (as described in (2)).

Similar to Problem 2, the solution $w^* = (w_l^*, w_h^*)$ to this problem satisfies the first order conditions with respect to w_h and w_l and four complementary slackness conditions corresponding to the four constraints. Wage schedule (6) satisfies these first order and complementary slackness conditions and is therefore the equilibrium wage. As before, uniqueness follows from Lemma 4. □

Proof of Theorem 1.

Proof. Follows immediately from Lemmas 2 and 3. □

The following Lemma offers a simple sufficient condition for uniqueness of the solution to a linear program, which is used in the proofs of Lemmas 1 and 3.

Lemma 4. *Consider the following linear program:*

$$\min_{x \in R^n} a'x \qquad \textit{subject to}$$

$b_j'x \geq c_j, \quad j = 1, .., J.$

The solution x^* *(which satisfies the first order condition* $a = \sum_j \lambda_j b_j$ *and the complementary slackness condition* $\lambda_j(b_j'x^* - c_j) = 0$, $\lambda_j \geq 0$*) is unique if (1) there are* n *indices* j *such that* λ_j *is not zero, and (2) the corresponding* b_j*'s are linearly independent.*

Proof. Suppose, on the contrary, there exists another solution $x \neq x^*$. By complementary slackness, $Bx^* = c$ where B is a matrix with rows b_j corresponding to the nonzero λ_j's. By (1) and (2), B is invertible, implying that $Bx \neq c$ (otherwise, we would have $x = x^*$). Furthermore, $a'x - a'x^* = \sum \lambda_j b_j'x - \lambda_j b_j'x^* = \sum \lambda_j(b_j'x - c_j) = 0$ since by feasibility all terms are nonnegative and by $Bx \neq c$ some term must be strictly positive. Therefore x is dominated by x^* contradicting the optimality of x. □

Table 1: Numerical Illustration of Possible Equilibria.

The numerical examples reported in this table illustrate that, depending on parameter values, any or all of the three sets of strategies (VP, TS, and NS, defined in Theorem 1 and in Lemma 1) can form an equilibrium. Each line corresponds to a separate example. All of the examples also assume $q_c = 3/4$, $q_o = 1$, $W = 1$, $u_o = 7/6$, $u_c = 1$, $a_l = 10$, $a_h = 15\frac{1}{6}$, and each line specifies the remaining parameters π_c, f_o, and f_c and reports the principal's payoffs Π_o from hiring only type o, Π_c from hiring only type c, and Π_{ns} from hiring both types. Note that all of the assumed parameters satisfy assumptions 1a - 1d). As reported in the last column of the table, the resulting equilibria are those that correspond to the highest values of the principal's payoff.

Parameters			Principal's Payoff			
π_c	f_o	f_c	Π_{vp}	Π_{ts}	Π_{ns}	Equilibria
1/6	13/50	1/2	11.78	11.75	11.75	VP
2/15	1/4	5/8	11.75	12.3125	11.75	TS
1/2	1/4	1/2	11.75	11.75	11.9375	NS
1/10	1/4	1/2	11.75	11.75	11.6875	VP and TS
2/5	1/4	3/8	11.75	11.1875	11.75	VP and NS
7/15	1/8	1/2	11.375	11.75	11.75	TS and NS
1/5	1/4	1/2	11.75	11.75	11.75	VP, TS and NS

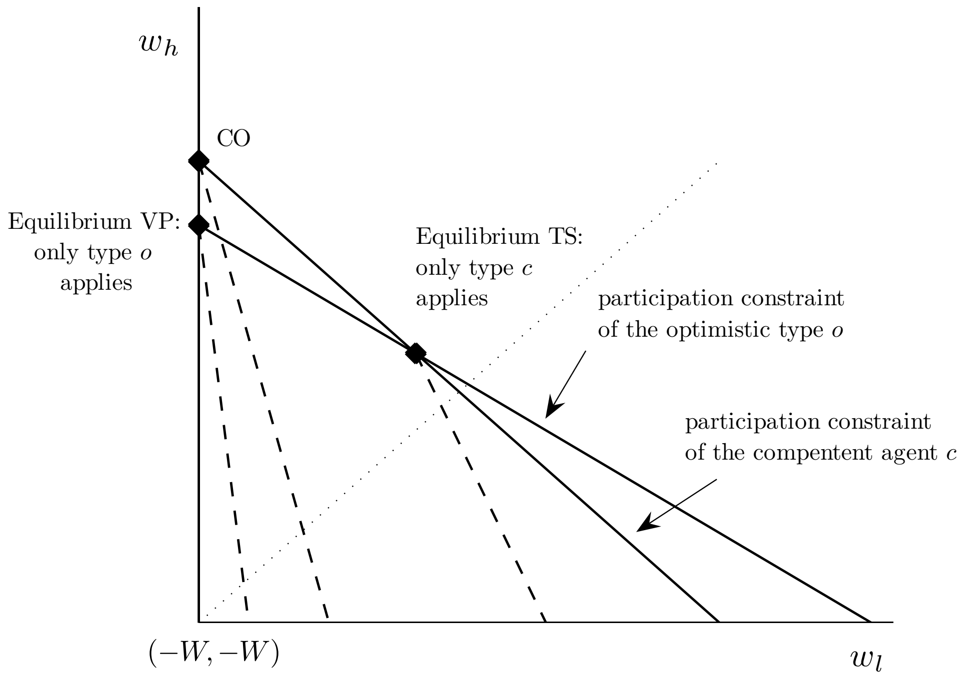

Figure 1: Model Analysis. The graph is built using the following parameter assumptions: $\pi_o = 0.9$, $u_o = 2.1$, $u_c = 2$, $q_o = 0.6$, $q_c = 0.5$, $f_o = 0.1$, $f_c = 0.3$, $W = 1$, $a_h - a_l = 10$. Given these parameters, there is only one separating equilibrium, Equilibrium VP (the optimistic agent is hired). If we instead assume $W = 0.5$, then there is a unique separating equilibrium, Equilibrium TS (the competent agent is hired). The corresponding figure is omitted since it is identical to the one shown, except that the axes move up and to the right so that they intersect at -0.5 rather than at -1.

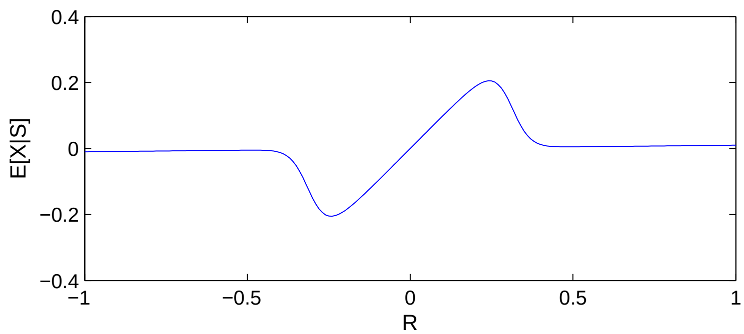

Figure 2: Principal's Posterior Given Manager's Forecast.

The graph shows the principal's posterior belief $E[X|R]$ given the manager's report R. When the report R is small, the principal believes that the manager is likely to be the competent type c, whose report R is close to the true value of X. For larger values of R, however, the principal believes that the manager is more likely to be type o who tends to overstate the true value of X. Thus, the line representing $E[X|R]$ starts close to a 45° line for small values of R, and becomes flatter for large values of R. The graph assumes $\pi_o = 0.2$, $\sigma_x = 0.1$, $\sigma_o = 10$, $\sigma_c = 1$, $K_o = 100$, and $K_c = 1$.

References

Adrian, Tobias, and Mark M. Westerfield. 2008. "Disagreement and Learning in a Dynamic Contracting Model." Federal Reserve Bank of New York Staff Report, 269.

Ben-David, John R. Graham, and Campbell R. Harvey. 2007. "Managerial Overconfidence and Corporate Policies." NBER working paper W13711.

Bhattacharya, S., and P. Pfleiderer. 1985. "Delegated Portfolio Management." *Journal of Economic Theory*, 36(1): 125.

Dybvig, P. H., H. K. Farnsworth, and J. N. Carpenter. 2004. "Portfolio Performance and Agency." NYU, Law and Economics Research Paper 04-010.

Gervais, Simon, J.B. Heaton, and Terrance Odean. 2007. "Overconfidence, Investment Policy and Manager Welfare."
http://www.bus.emory.edu/finseminars/Papers/Simon%20Gervais%20Paper.pdf.

Koszegi, Botond, and Wei Li. 2008. "Drive and Talent." *Journal of the European Economic Association*, 6(1): 210236.

Khorana, Ajay , Henri Servaes, and Lei Wedge. 2007. "Portfolio manager ownership and fund performance." *Journal of Financial Economics*, 85: 179204.

Malmendier, Ulrike, Geoffrey Tate. 2005. "Does Overconfidence Affect Corporate Investment? CEO Overconfidence Measures Revisited." *European Financial Management*, 11(5): 649659.

Ross, Stephen A. 1977. “The Determination of Financial Structure: the Incentive - Signalling Approach.” *Bell Journal of Economics*, 8(1): 23-40.

Santos-Pinto, Luis, Joel Sobel. 2005. “A Model of Positive Self-Image in Subjective Assessments.” *American Economic Review*, 95(5): 1386-1402

Spence, Michael. 1973. “Job Market Signaling.” *Quarterly Journal of Economics*, 87(3): 355374.

Van den Steen, E. 2004. “Rational Overoptimism (and Other Biases)” *American Economic Review*, 94(4): 1141-1151.

Van den Steen, E. 2005. “Organizational Beliefs and Managerial Vision.” *Journal of Law, Economics, and Organization*, 21(1): 256-283.