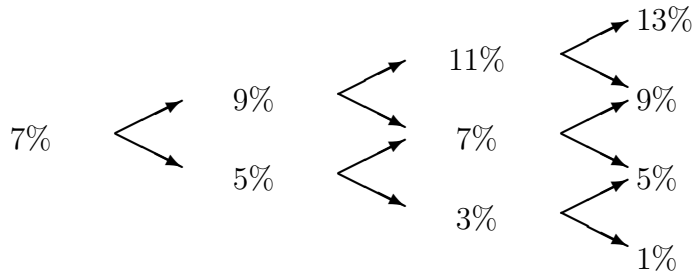
Practice problems for Lecture 5: answers

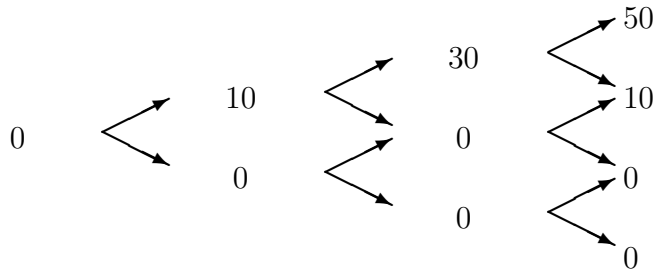
1. (more periods) Consider a three-year binomial model. The short riskless interest rate starts at 7% and moves up or down by 2% each period (i.e. up to 9% and down to 5% in the second year). The artificial probability of each of the two states at any node is 1/2.

a. What is the fair price now of a 3-year cap with notional $\$1,000,000$ and cap rate of 8%? (Reminder: a cap pays at each point in time the notional times the excess, if any, of the reference rate (here the riskless rate) above the cap rate at each point of time at and before maturity.)

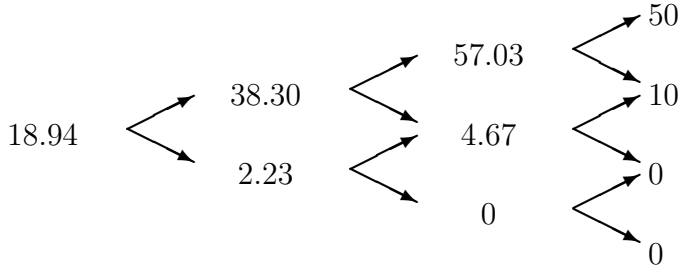
The rate tree is



Cash flows ($\$1,000$'s) are

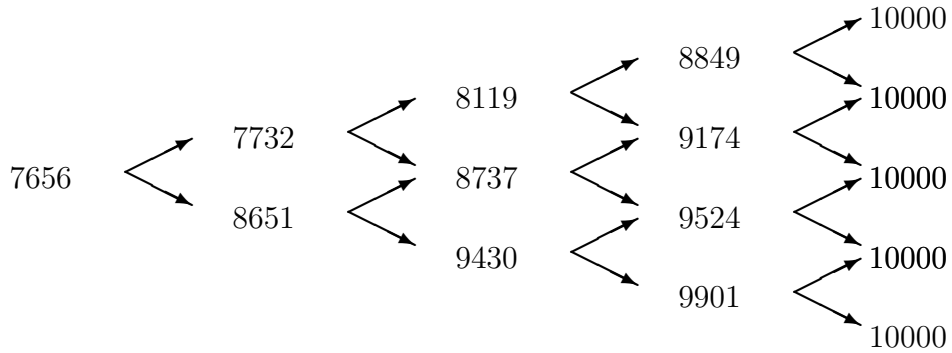


Prices ($\$1,000$'s, measured before cash flows are received) are



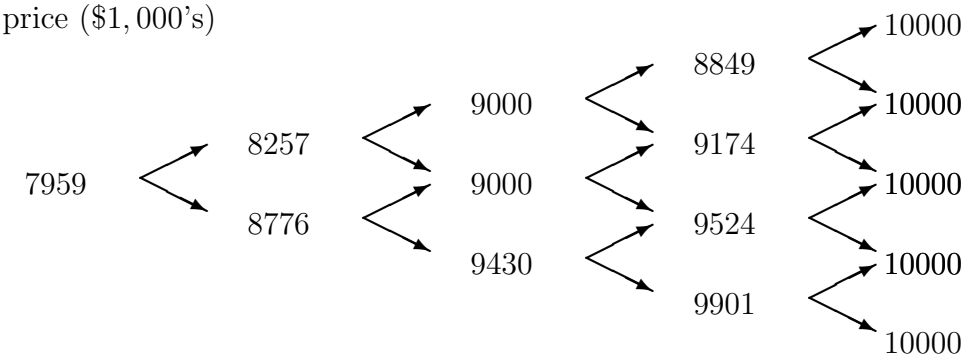
b. Draw the price tree of a zero coupon bond with face value $\$10,000$ maturing in 4 years. What is its present value?

price ($\$1,000$'s)



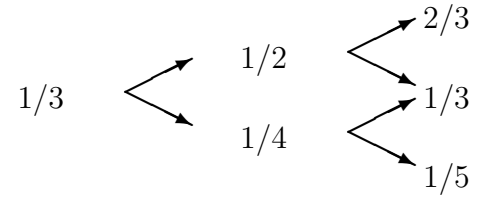
c. Suppose the zero coupon bond in (b) is puttable two years from now at $\$9,000$, that is you will have an option to sell the bond back to the firm for $\$9,000$ at that point of time. What is the price of the bond at every node? Compare your answer with (b). (Hint: the price of the bond at a node where it can be put is the larger of $\$9,000$ and it what it is worth if not put. Another hint: the possibility of conversion affects the value all the way backwards in the tree, not just at that time.)

price ($\$1,000$'s)

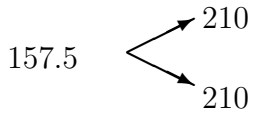


The option to put the bond makes it more valuable.

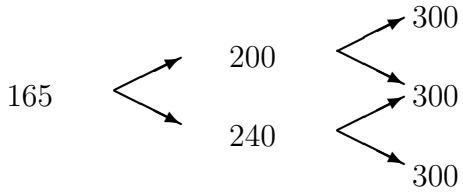
2. (put-call parity) Consider a binomial model with an annual interest rate tree



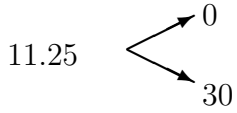
a. What is the price at each node of a discount bond with face value \$210 and maturity one year from now?



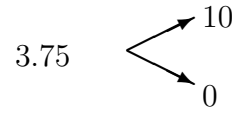
b. What is the price at each node of a discount bond with face value \$300 and maturity two years from now?



c. What is the price at each node of a European call option on the two-period bond in part b. with a maturity one year from now and strike equal to \$210?



d. What is the price at each node of a European put option on the two-period bond in part b. with a maturity one year from now and strike equal to \$210?



e. For stock options, put-call parity says $S + P = B + C$ or equivalently $S + P - B - C = 0$, where P and C are a put and a call on the same underlying stock S having the same maturity and strike, and B is the price of a discount bond maturing with the option and having face value equal to the common strike price. Verify put-call parity in this problem, taking the two period bond playing the part of the stock.

$$\begin{aligned} S + P - B - C &= 165 + 3.75 - 157.5 - 11.25 \\ &= 168.75 - 168.75 \quad \text{Confirmed!} \end{aligned}$$