FIN 550 Exam, June 25, 2008

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This is a closed-book examination. Answer all questions as directed. Mark your answers directly on the examination. Make sure each answer is clearly indicated. There are no trick questions on the exam. Good luck!

1. True-False (24 points)

A. Linear programs typically have interior solutions.

B. A local optimum of a convex optimization problem is a global optimum.

C. The Fundamental Theorem of Asset Pricing is about the absence of arbitrage.

D. A grid search is faster but less reliable than the Newton method.

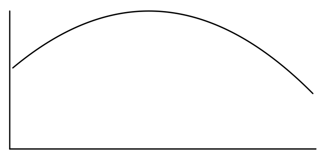
E. The value function in a dynamic program is the value of continuing using the optimal strategy.

F. Slack variables are used to convert inequality constraints into equality constraints.

2. Multiple choice – Univariate optimization (16 points)

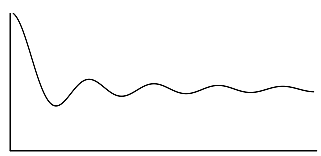
Choose the answer that best describes each picture

A.



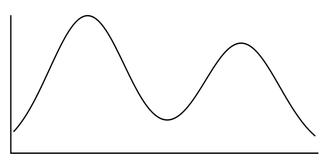
(i) strictly convex function
(ii) strictly concave function
(iii) linear function
(iv) none of the above

B.



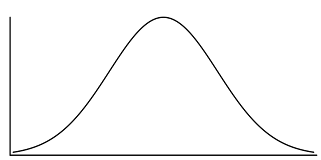
(i) strictly convex function
(ii) strictly concave function
(iii) linear function
(iv) none of the above

C.



(i) no local maximum
(ii) one local maximum
(iii) two local maxima
(iv) three local maxima

D.



(i) no local maximum
(ii) one local maximum
(iii) two local maxima
(iv) three local maxima

3. Linear Programming (30 points)

Consider the following linear program:

Choose nonnegative x_1, x_2, and x_3 to
maximize $2x_1 + x_2 + 3x_3$, subject to
$x_1 + 2x_2 + 3x_3 \leq 6$ and
$x_1 + x_2 \leq 3$

A. What is the dual linear program?

B. Is the primal feasible? Is the dual feasible?

C. Infer from the answers in part B: Is the primal bounded? Is the dual bounded?

D. Solve the dual problem.

E. Use the solution to the dual problem to solve the primal problem.

4. Kuhn-Tucker Conditions (30 points)

Consider the following optimization problem:

Choose c_u and c_d to
maximize $\frac{1}{2}\log(c_u) + \frac{1}{2}\log(c_d)$, subject to
$\frac{4}{5}(\frac{1}{4}c_u + \frac{3}{4}c_d) \leq 6$.

This is a single-period choice of investment for consumption in a binomial model with log utility, initial wealth of 6, actual probabilities 1/2 and 1/2, risk-neutral probabilities 1/4 and 3/4, and riskfree rate of 25% (and therefore discount factor 4/5).

A. What are the objective function, choice variables, and constraint?

B. What are the Kuhn-Tucker conditions?

C. If we add constraints $c_u \geq 6$ and $c_d \geq 6$, what are the Kuhn-Tucker conditions now?

5. Bonus question (30 bonus points)

A. Solve the optimization problem in problem 4 without the extra constraints in part 4C.

B. Solve the optimization problem in problem 4 with the extra constraints in part 4C.

Useful formulas

For the problem

Choose $x \in \Re^N$ to
maximize $f(x)$
subject to $(\forall i \in \mathcal{E}) g_i(x) = 0$, and
$(\forall i \in \mathcal{I}) g_i(x) \leq 0$.

The Kuhn-Tucker conditions are

$\nabla f(x^*) = \sum_{i \in \mathcal{E} \bigcup \mathcal{I}} \lambda_i \nabla g_i(x^*)$
$(\forall i \in \mathcal{I}) \lambda_i \geq 0$
$\lambda_i g_i(x^*) = 0$

Choose $x \geq 0$ to
minimize $c^\top x$
subject to $Ax \geq b$

there is a dual LP

Choose $y \geq 0$ to
maximize $b^\top y$
subject to $A^\top y \leq c$