Problem Set 3: Linear Programming Duality and Fundamental Theorem of Asset Pricing
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At the start of class next week, submit only problem 3 for grading.

1. Dual LPs

A. Write down the dual LP for the following LPs

Choose nonnegative x_1 and x_2 to
minimize $x_1 + 3x_2$, subject to
$x_1 + x_2 \geq 4$.

Choose nonnegative x_1 and x_2 to
minimize $x_1 - 2x_2$, subject to
$x_1 \geq 1$ and
$x_1 + 2x_2 \geq 2$.

B. For each primal and dual LP, answer the following. Is the LP feasible? Is the LP bounded? If the LP has an optimal solution, what is it?

C. Discuss the results in terms of the duality theorems.

2. Finding arbitrage: puts

You can buy HAL stock for \$59.36/share or go short at \$59.32 and you can also buy or sell listed puts all maturing on the same date in August (these are all-in prices based on most recent trades plus an estimate of half the spread plus trading costs):

strike	put ask	put bid
45	1.52	1.48
50	2.77	2.73
55	2.99	2.85
60	5.95	5.85
65	10.65	10.35

In addition, the riskless borrowing rate for the maturity of the options is 1% simple interest and the lending rate is 0.5% simple interest.

A. Set up the state-space tableau for short and long positions in the put options and the underlying stock and riskfree borrowing/lending. Calculate the payoffs based on terminal stock prices 0, 45, 50, 55, 60, 65, and 1000 (all the strike prices plus two extreme values).

For this analysis, assume these are European options.

B. Write down a linear programming problem that searches for an arbitrage given these trading opportunities.

C. Use Solver to search for an arbitrage opportunity.

D. Examine the optimal solution found by Solver and describe it qualitatively.

3. Finding arbitrage: calls

You can buy HAL stock for $59.36/share or go short at $59.32 and you can also buy or sell listed calls all maturing on the same date in August (these are all-in prices based on most recent trades plus an estimate of half the spread plus trading costs):

strike	call ask	call bid
45	13.24	13.13
50	12.72	12.63
55	9.41	9.28
60	7.31	7.17
65	6.61	6.57

In addition, the riskless borrowing rate for the maturity of the options is 1% simple interest and the lending rate is 0.5% simple interest.

A. Set up the state-space tableau for short and long positions in the call options and the underlying stock and riskfree borrowing/lending. Calculate the payoffs based on terminal stock prices 0, 45, 50, 55, 60, 65, 70, and 1000 (all the strike prices plus two extreme values).

For this analysis, assume these are European options.

B. Write down a linear programming problem that searches for an arbitrage given these trading opportunities.

C. Use solver to search for an arbitrage opportunity.

D. Examine the optimal solution found by Solver and describe it qualitatively.