FIN 550 Exam

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This is a closed-book examination. You may not use texts, notes, a crib sheet, calculator, cell phone, listening device, or any other electronics. Answer all questions as directed on the Blue Books provided. Be sure your answers are clearly marked. There are no trick questions on the exam. Good luck!

0. PLEDGE

The work on this exam is my own alone, and I have conformed with the rules of the exam and the code of the conduct of the Olin School.

Signed name ____

Printed name (write clearly) ____

1. True-False (25 points)

A. Constrained problems never have interior solutions.

B. In an unconstrained problem with a concave objective function, a local maximum is a global maximum.

C. Multiplying a random variable by a constant greater than 1 increases its kurtosis.

D. The eigenvalues of a positive definite matrix are all positive.

E. A linear program with a bounded dual always has an optimal solution.

2. Linear Programming (25 points) Consider the following linear program:

Choose nonnegative x_1, x_2, and x_3 to
maximize $x_1 + 6x_2 + 3x_3$, subject to
$x_1 + 3x_2 + 3x_3 \leq 6$ and
$x_1 + x_2 \leq 3$

A. What is the dual linear program?

B. Solve the dual problem.

C. Use the solution to the dual problem to solve the primal problem.

D. Show that the strong duality theorem holds in this example.

3. REGIME-SWITCHING (25 points) Consider a two-state Markov Chain in continuous time. Regime switches take place at the following rates:

state 1 $\rightarrow$ state 2	probability 0.10/year
state 2 $\rightarrow$ state 1	probability 0.15/year

Initially (at time $t = 0$), we are in state 2.

A. What is the matrix A in the ODE

$$\pi'(t) = A\pi(t)$$

describing the dynamics of the vector $\pi(t)$ of future regime probabilities?

B. Solve for the eigenvalues of A.

C. Solve for the associated eigenvectors.

D. Write down the general solution of the ODE.

E. Write down the particular solution corresponding to the initial condition that we start in state 2.

F. A project costing $100,000 has a cash flow of $10,000/year in state 1 and $1,000/year in state 2. The cash flows continue forever. If the continuously-compounded interest rate is 5%/year, does this project have a positive expected NPV?

4. Kuhn-Tucker Conditions (25 points)

Consider the following optimization problem:

Choose c_u and c_d to
maximize $\frac{2}{3}\log(c_u) + \frac{1}{3}\log(c_d)$, subject to
$\frac{4}{5}(\frac{1}{2}c_u + \frac{1}{2}c_d) = 6$,
$c_d \geq 6$, and $c_u \geq 6$.

This is a single-period choice of investment for consumption in a binomial model with log utility, initial wealth of 6, actual probabilities 2/3 and 1/3, risk-neutral probabilities 1/2 and 1/2, and riskfree rate of 25% (and therefore discount factor 4/5).

A. What are the objective function, choice variables, and constraints?

B. What are the Kuhn-Tucker conditions?

C. Solve the optimization problem.

5. Bonus problem (30 bonus points)

Consider the Markov switching model in problem 3 and in particular the project described in part 3E. Suppose that instead of starting the project immediately, we can pay $3,000 per unit time to keep the project alive without undertaking it. To get any credit for this problem, you must show your work, and there is no use just guessing.

A. What is the optimal strategy? You can restrict your analysis to the only sensible strategies (a) abandon the project, (b) invest immediately, and (c) spend 3,000 per unit time to keep the project alive until we arrive in state 1, and at that time invest in the project.

B. What is the NPV of the optimal strategy?

Useful formulas

For the problem

Choose $x \in \Re^N$ to
maximize $f(x)$
subject to $(\forall i \in \mathcal{E}) g_i(x) = 0$, and
$(\forall i \in \mathcal{I}) g_i(x) \leq 0$,

the Kuhn-Tucker conditions are

$\nabla f(x^*) = \sum_{i \in \mathcal{E} \bigcup \mathcal{I}} \lambda_i \nabla g_i(x^*)$
$(\forall i \in \mathcal{I}) \lambda_i \geq 0$
$\lambda_i g_i(x^*) = 0.$

For the LP

Choose $x \geq 0$ to
minimize $c^\top x$
subject to $Ax \geq b$,

the dual LP is

Choose $y \geq 0$ to
maximize $b^\top y$
subject to $A^\top y \leq c$.