Problem Set 3: LPs, Duality, and Fundamental Theorem of Asset Pricing
FIN 550: Numerical Methods and Optimization in Finance
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At the start of class November 27, submit Problem 2 and Problem 4.

1. Dual LP

A. Write down the dual LP for the following LP

Choose nonnegative x_1, x_2, and x_3 to
minimize $x_1 - 2x_2 + 3x_3$, subject to
$x_1 \geq 1$ and
$x_1 + 2x_2 + 4x_3 \geq 2$.

B. For the primal and dual LPs, answer the following. Is the LP feasible? Is the LP bounded? If the LP has an optimal solution, what is it?

C. Discuss the results in terms of the theorem about feasibility and boundedness of the primal and dual.

2. Dual LP

A. Write down the dual LP for the following LP

Choose nonnegative x_1, x_2, and x_3 to
minimize $3x_1 - 3x_2 + 2x_3$, subject to
$x_1 + x_2 + 3x_3 \geq 4$
$x_1 + x_2 + x_3 \leq 3$.

B. For the primal and dual LPs, answer the following. Is the LP feasible? Is the LP bounded? If the LP has an optimal solution, what is it?

C. Discuss the results in terms of the theorem about feasibility and boundedness of the primal and dual.

3. Finding arbitrage: puts

You can buy HAL stock for $59.36/share or go short at $59.32 and you can also buy or sell listed puts all maturing on the same date in August (these are all-in prices based on most recent trades plus an estimate of half the spread plus trading costs):

strike	put ask	put bid
45	1.52	1.48
50	2.77	2.73
55	2.99	2.85
60	5.95	5.85
65	10.65	10.35

In addition, the riskless borrowing rate for the maturity of the options is 1% simple interest and the lending rate is 0.5% simple interest.

A. Set up the state-space tableau for short and long positions in the put options and the underlying stock and riskfree borrowing/lending. Recall that the payoff of a put option is $max(X - S, 0)$, where X is the strike price and S is the stock price. Calculate the payoffs based on terminal stock prices 0, 45, 50, 55, 60, 65, and 1000 (all the strike prices plus two extreme values).

For this analysis, assume these are European options.

B. Write down a linear programming problem that searches for an arbitrage given these trading opportunities.

C. Use a computer to search for an arbitrage opportunity (or do it by hand if you prefer).

D. Examine the optimal numerical solution and describe it qualitatively.

4. Finding arbitrage: calls

You can buy HAL stock for \$64.3/share or go short at \$63.2 and you can also buy or sell listed calls all maturing on the same date in February (these are all-in prices based on most recent trades plus an estimate of half the spread plus trading costs):

strike	call bid	call ask
45	21.20	21.50
50	16.50	17.00
55	12.50	13.00
60	10.50	11.00
65	3.50	4.00

In addition, the riskless borrowing rate for the maturity of the options is 1% simple interest and the lending rate is 0.5% simple interest.

A. Set up the state-space tableau for short and long positions in the call options and the underlying stock and riskfree borrowing/lending. Calculate the payoffs based on terminal stock prices 0, 45, 50, 55, 60, 65, 70, and 1000 (all the strike prices plus two extreme values).

For this analysis, assume these are European options and that there are no dividends.

B. Write down a linear programming problem that searches for an arbitrage given these trading opportunities.

C. Use a computer to solve the linear program (or do it by hand if you prefer).

D. Examine the optimal numerical solution and describe it qualitatively.

E. Explain how the analysis would change if there is an unknown dividend.

5. (extra for experts) Putting in an extreme stock price at the high end only approximates the condition for having no arbitrage for sufficiently high prices. What is the exact condition and how can it be implemented in the search for an arbitrage?