FIN 550J Exam, October 21, 2010

Phil Dybvig

This is a closed-book examination. You may not use texts, notes, a crib sheet, calculator, cell phone, listening device, or any other electronics. Answer all questions as directed. Make sure each answer is clearly indicated.

0. PLEDGE

The work on this exam is my own alone, and I have conformed with the rules of the exam and the code of the conduct of the Olin School.

Signed name ____

Printed name (write clearly) ____

1. PROBABILITIES (20 points) Let the stock price S take on values uniformly distributed on $[40, 60]$. A digital option on the stock has payoff

$$D = \begin{cases} 1 & \text{for } S > 55 \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

a. What is the density (pdf) of the stock price S?

b. What is the distribution function (cdf) of the stock price S?

c. What is the distribution function (cdf) of the payoff D of the digital option?

d. Compute the mean, variance, and standard deviation of the digital option payoff.

2. LINEAR EQUATIONS (24 points) Consider the system of equations:

$$x_1 = 17 - 2x_2 - 3x_3$$
$$x_2 = 8 - 2x_1 - x_3$$
$$x_3 - 5 = x_1 - x_2$$

a. Write these equations in the form $Ax = b$. What are A and b?

b. Solve for x using Gaussian elimination.

3. OPTIMIZATION (26 points) Solve the following maximization problem:

Choose x_1 and x_2 to
maximize $10x_1 + 5x_2 - x_1^2 - x_1x_2 - x_2^2$
subject to $x_2 \leq 5$

Note: the second-order conditions are satisfied because the Hessian of the objective function is negative definite and the constraint set is convex. You do not need to prove this.

4. EIGENVALUES AND EIGENVECTORS (30 points) Let

$$C = \begin{pmatrix} .5 & .4 \\ .5 & .6 \end{pmatrix}$$

A. Compute the eigenvalues of C.

B. Compute the corresponding eigenvectors of C.

C. Use the eigenvalues and eigenvectors to compute $C^5(0,1)^T$.