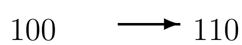
FIN 451 Exam, November 8, 2007

Phil Dybvig

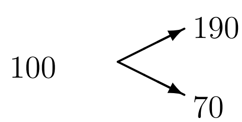
This is a closed-book examination. Answer all questions as directed. Mark your answers directly on the examination. On the valuation question, make sure your answer is clearly indicated. There are no trick questions on the exam. Good luck!

A. Binomial Option Pricing in One Period *40 points*

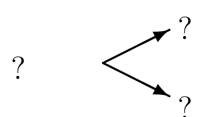
Riskless bond (interest rate is 10%):



Stock:

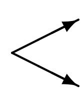


European call with a strike price of 124:



Actual probabilities: 1/2 and 1/2.

1. What are the payoffs of the European call in the up and down states?



2. What are the risk-neutral probabilities for the two states tomorrow?

3. What is the price of the European call today?

4. What is the portfolio of stocks and bonds that replicates the call?

B. Concepts (multiple choice) *20 points*

1. If we buy a call option, we are

a. long the stock and short volatility.
b. long the stock and long volatility.
c. short the stock and short volatility.
d. short the stock and long volatility.

2. If we sell a put option, we are

a. long the stock and short volatility.
b. long the stock and long volatility.
c. short the stock and short volatility.
d. short the stock and long volatility.

3. Which of the following is the best descripotion of the payoff (per dollar of underlying) from holding stock index futures?

a. return on the underlying portfolio
b. return on the underlying portfolio less the riskfree rate
c. return on the underlying portfolio less the riskfree rate, plus dividends
d. return on the underlying portfolio less the riskfree rate, less dividends

4. (put-call parity) For European options, buying a call is equivalent to

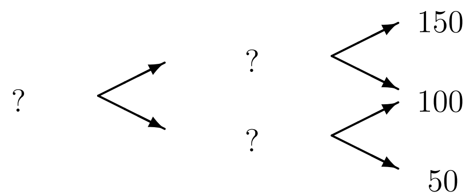
a. selling a put, selling the stock, and buying the bond
b. buying a put, selling the stock, and buying the bond
c. selling a put, buying the stock, and selling the bond
d. buying a put, buying the stock, and selling the bond

5. In the Orange County financial crisis, huge amounts of money were lost betting on

a. gold prices
b. orange juice futures
c. interest rates
d. the stock market

C. Binomial Futures Option Pricing *40 points*

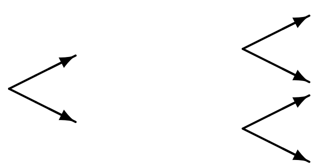
Two periods from now, an index futures contract matures and we model its price using a binomial model that takes on a value of \$150, \$100, or \$50:



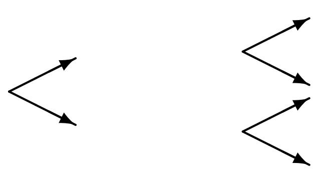
The short riskless interest rate is 25% per period. The risk-neutral probabilities of up and down moves are equal at 1/2, while the actual probability of an up move is 3/5 and the actual probability of a down move is 2/5.

Consider European and American futures call options with a strike price of \$75 and maturity two periods from now.

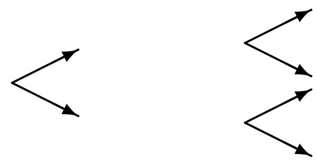
1. What are futures prices at each node in the tree?



2. What are the European call option values at each node?



3. What are the American call values at each node?



4. (conceptual question/short answer) In this problem the risk-neutral probabilities are not equal to the actual probabilities. How can that make sense?

D. Bonus question *20 points*

Price a claim which pays the square root of the final stock price n periods from now. The initial stock price is S_0, and in each period the stock price goes up by a factor u with risk-neutral probability 1/2 or down by a factor d, also with risk-neutral probability 1/2. One plus the riskfree rate is r at all nodes.

Useful Formulas

Binomial model: if the stock has up and down factors u and d and one plus the riskfree rate is r, then the risk-neutral probabilities are

$$\pi_u = \frac{r-d}{u-d}$$

$$\pi_d = \frac{u-r}{u-d}$$

and the one-period option valuation is

$$\text{price} = \frac{1}{r}\left(\pi_u V_u + \pi_d V_d\right).$$

The Black-Scholes call price is

$$C(S,T) = SN(x_1) - BN(x_2),$$

where S is the stock price, $N(\cdot)$ is the cumulative normal distribution function, T is time-to-maturity, B is the bond price $Xe^{-r_f T}$, r_f is the continuously-compounded riskfree rate, σ is the standard deviation of stock returns,

$$x_1 = \frac{\log(S/B)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T},$$

and

$$x_2 = \frac{\log(S/B)}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T}.$$

Note that $\log(\cdot)$ is the natural logorithm.

The Black-Scholes call price can be approximated by

$$\frac{S-B}{2} + .4\frac{S+B}{2}\sigma\sqrt{T}.$$

The put-call parity formula is

$$B + C = S + P.$$