Portfolio Performance and Agency

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Abstract

The evaluation and compensation of portfolio managers is an important practical problem. Optimal compensation will induce managers to expend effort to generate information and to use it appropriately in an informed portfolio choice. Our general model points the way towards analysis of optimal performance evaluation and contracting in a rich model. Optimal contracting in the model includes an important role for portfolio restrictions that are more complex than the sharing rule. The manager’s compensation includes an incentive fee based approximately on the excess of return above an endogenously chosen benchmark. This measure is often used by practitioners but is simpler than the Jensen measure and other measures commonly recommended in the academic literature. In addition to the excess return above the fixed benchmark, the manager is given some additional incentive to take a position that deviates from the benchmark. Efficient contracting involves restrictions on what portfolio strategies can be pursued, or equivalently, prior communication of the information gathered.

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I Introduction

The appropriate evaluation and compensation of portfolio managers is an ongoing topic of debate among practitioners and regulators. Although performance measurement and optimal managerial contracting are two sides of the same coin, the academic literature has largely considered the two questions separately. Typically, performance measurement has been studied in the context of models with realistic security returns but no consideration of the incentives created by the measure. And, optimal contracting has been studied in information models with careful consideration of incentives but simplistic models of portfolio choice and security returns. This paper takes an optimal contracting approach but uses a rich model of security returns with many states of nature and full spanning over states distinguished by stock returns. The optimal contract restricts the manager’s choice of portfolio, as do investment guidelines in practice.

This paper treats the contracting between investor and manager as an agency problem. In a traditional agency problem, the agent expends costly effort on behalf of the principal. In a portfolio agency problem, the manager (the agent) expends costly effort to generate an informative signal that is correlated with future asset returns and then uses the signal to make an informed portfolio choice on behalf of the investor (the principal). The solution to the traditional agency problem features a trade-off between incentives and risk-sharing. The solution to the portfolio agency problem also trades off incentives and risk-sharing, but the incentives are not only for effort but also for choosing the intended portfolio as a function of the realized signal.

Some papers on portfolio agency assume that a sharing rule for portfolio profits, similar to the sharing rule for output in traditional agency problems, is the full managerial contract. Unfortunately, a simple sharing rule alone is not rich enough to implement the optimal contract, and it turns out a better contract is available that requires the manager to reveal the signal before portfolio returns are realized. To make sure we capture all possible

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1 See Ross (1973). For a survey of the agency literature see Stole (1993).
2 Admittedly, it may not be realistic to assume that portfolio managers have the ability to outperform the market, since attempts to document consistent superior performance have failed. However, we need some assumption along these lines if we are going to rationalize significant fees for active management, and at least some managers and their clients believe superior performance is possible.
institutions in our model, we can rely on the *revelation principle*\(^3\), which says that all possible outcomes that can be implemented by any mechanism can be implemented by *direct mechanisms*. In a direct mechanism, the manager is asked to report the signal and then the action (the portfolio choice in our case) is implemented by a third party according to a pre-agreed rule. It does not matter that this mechanism is not a literal description of what actually happens, since there are equivalent alternative mechanisms. For example, constraining the set of portfolio strategies to the ones selected by the direct mechanism is equivalent to having the manager report the signal with implementation by a third party. The important thing is that using a direct mechanism ensures us of having a sufficiently general set of contracts.

In solving the contracting problem, we work with utilities as choice variables as in Grossman and Hart (1983). In that paper, risk neutrality of the principal and using agent utilities as choice variables converts the agency problem into a linear program. For us, the risk aversion of the principal implies that the transformed problem has a concave objective function, which is the principal’s indirect utility given the budget share and the signal state. For most of the paper we adopt the usual first-order approach of replacing the incentive-compatibility constraints by their first-order conditions. However we must emphasize that this approach is not always correct, and we provide numerical results for an example with a non-locally binding incentive compatibility constraint.

While our model allows rich sets of contracts, portfolio strategies, return distributions, and signals, we do make a number of simplifying assumptions for tractability. One important assumption is that effort operates through the mixing probability between models in which the signal is relatively more or less informative; this assumption has been described previously by Holmström (1984) and Rogerson (1985). Another important assumption is the use of logarithmic utility. Logarithmic utility for the principal allows us to write down explicitly the indirect utility for the principal given the signal state and the principal’s conditional budget share. Assuming logarithmic utility is an extension for the agency literature (which often assumes that the principal is risk-neutral), but we know from the literature on portfolio choice how special this assumption is. Another simplifying assumption made in the specific numerical calculations is that pricing is as in the Black-Scholes model, and the joint distribution between the log market return and the signal taken to

\(^3\)See Holmström (1978) and Myerson (1979).
be joint Gaussian in each of two latent mixing states.

Our analysis considers three cases. The first-best is the traditional competitive solution that obtains when effort and signal states are publicly observable. Because markets are complete and we have logarithmic utility, we know explicitly the payoffs and utilities in the first-best, at least conditional on the level of effort. The second-best adds a constraint that the manager must have correct incentives for producing the appropriate level of effort under the assumption that the signal is publicly observed and can be verified. In this case, the first-order approach (of replacing incentive constraints by their first-order conditions) is fully justified because we are using a mixture model, and we have a full theoretical characterization of the solution. The main theoretical result in this case is that the optimal contract gives the manager a proportional share of the portfolio with an additional tilt into the excess return of the portfolio over an endogenous benchmark. In the third-best, the signal is not publicly observed and we must give the manager the incentive not only to expend the desired effort but also to choose the appropriate action given the signal observed by the manager. In our numerical examples, the third-best solution is similar to the second-best but contains adjustments that prevent the manager from gaming the contract.

For example, the third-best contract must make sure that the manager does not have the incentive to be a “closet indexer.” In industry parlance, the term “closet indexer” is applied to managers who adopt a passive strategy like an indexer but collects fees appropriate for active management. A closet indexer might expend no effort, collect the management fee, and choose a noncommittal portfolio near the benchmark. In fact, the optimal contract constrains the manager to a set of contracts that gives a small fee to a manager choosing a relatively safe portfolio. The fee schedule also penalizes such a manager if the market goes way up or way down, which would be an unlikely outcome when an informed manager gets a neutral signal. In the third-best, a manager reporting an extreme signal receives extra compensation both directly with an additional budget share and indirectly with a reduction in risk (both compared to the second-best).

Conceptually, the current paper is very similar to Kihlstrom (1988), a previous analysis of portfolio agency using optimal contracting principles. Both papers have a model with the same basic structure. The amount of

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4 Some question the value of active management and suggest there is no justification for higher fees for active managers; we will not address this question.
costly effort expended by the portfolio manager determines the quality of information the manager will receive about subsequent security returns. The payoff on the incentive contract depends not only on the realized payoff on the portfolio, but also on the “reported” signal either explicitly (in a direct mechanism) or implicitly through the portfolio choice. Importantly, in both papers the manager does not have complete freedom in choosing the portfolio and must choose from a set of portfolios which corresponds to the set of possible signal states.

The most important difference between the current model and the model in Kihlstrom (1988) is in the richness of the set of states and signals. Our model allows for a large set of market states and a large set of signal states while the model in Kihlstrom (1988) has only two market states and two signal states. This is important because it admits a much richer set of candidate portfolio strategies and contracts (when there are two states all payoffs are linear). Furthermore, having many states admits locally binding constraints on the incentive compatibility of the use of information. By contrast, with only two signal states, there is only the issue of acting the opposite of what the signal indicates, and no possibility that the manager might want to take a position that is only slightly too risky or too conservative. In addition to this primary difference, there is also in the model of Kihlstrom (1988) linear utility for the investor, which implies a corner solution and would lead to the extreme behavior of plunging into the security with the largest conditional mean return if this assumption were maintained when moving to many states.

Another paper with similar goals is Stoughton (1993), which considers affine (linear plus a constant) and quadratic contracts in a two-asset world. Stoughton’s paper works in the traditional framework of an incentive contract characterized by a sharing rule with no restriction on portfolio choice. The paper contains a curious result, also derived later by Admati and Pfleiderer (1997) in a very similar model: varying the slope of an affine contract is

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5There is a conceptual problem with the analysis of the quadratic contract in Stoughton (1993). It is argued that one particular sequence of contracts achieves the first-best in a limit. However, the sense of convergence analyzed depends on the utility representation. Using a more reasonable sense of convergence measured by certainty-equivalent, the contracts do not approach first-best. Specifically, the manager’s utility is \( U_B(w) = -\exp(-bw) \) and therefore the certainty equivalent is the inverse of this function \( CE(u) = -\log(-u)/b \). Then we can compute from (29) and (30) in Stoughton (1993) (using also (4)), that the difference in certainty equivalents for small \( b \) is approximately \(-\log(1-2a\gamma_2/H)/(2a)\), which is a positive constant.
ineffective in producing incentives because the leverage in the contract can be undone by changing the leverage in the portfolio choice. For us, this result does not hold because the manager is not free to change the portfolio policy arbitrarily. This is part of why the general contracts we consider (where the portfolio chosen has to be consistent with what is planned for some signal) can do better than the traditional sharing rule with no restriction on portfolio choice.

Another related paper is Zender (1988), which provides a number of results. The Jensen measure is shown to be the optimal linear contract in a reduced-form model for a mean-variance world in which the expenditure of costly effort influences the conditionally efficient portfolio in a particular way. In the same model, it is shown that a Mirrlees (1974) forcing contract (with a small probability of extreme punishments when the realization is grossly incompatible with the signal) can get arbitrarily close to the first-best if we allow for a general nonlinear contract.\(^6\) Finally, an interpretation of the problem in terms of continuous-time agency model of Holmström and Milgrom (1987) is given. The main weaknesses of Zender (1988) are that the mapping from effort to efficient portfolio is a black box and that it is unclear what underlying model it is a reduced form for, or indeed whether the optimal contract in the reduced form is also optimal in the underlying model. It should be mentioned that Sung (1995) has a nontrivial extension of Holmström and Milgrom (1987) that includes control of variance as well as mean, and one of the applications of the general result is to a portfolio agency problem similar to the one described in Zender (1988).

It is useful to mention what our model does not try to do. For example, Bhattacharya and Pfleiderer (1985) use a screening model to study self-selection of portfolio managers, while we use an agency model to study incentives for managers to exert effort. Trying to screen managers by their own confidence levels may not be a good idea. Indeed, we do not want to hire a manager who is very confident about outperforming the market by 40% per year with essentially no risk, since we are likely to think such a manager is overconfident and naive about the functioning of the market. Another example of what our model does not do is to analyze career concerns, i.e., the effect of the manager’s portfolio performance on future salaries or funds under management and the impact this has on incentives. In effect we analyze a

\(^6\)There would presumably be a Mirrlees (1974) forcing contract in the model of Stoughton (1993) as well if it were extended to admit general nonlinear contracts.
manager in the last year before retirement with no way to sell the manager’s track record. Alternatively, our problem covers the case in which the career concerns are somehow neutralized. The model in this paper can be extended to include career concerns, and using the tools developed here to analyze that problem is a promising extension which would generate different (and possibly more realistic) portfolio constraints.

Although not a model of delegated portfolio management, the model of delegated expertise of Demski and Sappington (1987) bears several similarities to the model of this paper. In that paper, the agent chooses whether to incur costly effort in gathering information and then takes some action on behalf of the principal. Demski and Sappington point out that the principal must choose the optimal level of effort as well as an action for each possible realized signal. The main differences between that paper and this paper is that the principal is risk neutral and the sharing rule over the output depends only on output and not on the action taken (or the signal observed by the agent).

Section II describes the optimal contracting problem. Section III presents numerical results and shows the qualitative features of the first-best, second-best, and third-best contracts. Section IV discuss analytical results for these contracts. Section V presents the version of the model in which the incentive compatibility of effort constraint is remotely binding rather than locally binding. Section VI concludes.

II The General Problem

The general formulation of the problem is as follows. We assume that the market is complete over states distinguished by security payoffs. Let \( \omega \) denote such a state. We will refer to \( \omega \) as a market state and will denote by \( p(\omega) \) the price of a claim which pays a dollar in state \( \omega \).

The manager has ability in the form of costly information acquisition. The information gathered is in the form of a private signal \( s \) that is never revealed publicly. We think of the limit in which the manager is very small, and we assume that using the information has no effect on equilibrium prices. Also, because the signal is never made public it is not possible to write claims

\footnote{This paper also abstracts from the usual hierarchical structure of separate management of different asset classes, uncertainty about the manager’s preferences, and any difficulties in preventing the manager from undoing incentives through private portfolio trades.}
dependent on the value of the signal. The signal is more or less informative about the future market state depending on the action $a$ which indicates the amount of costly effort expended on information-gathering. We denote the joint distribution of market state and signal by $f(s, \omega; a)$.

Our formal game is set up in the tradition of contracting theory and may seem peculiar because it is not a literal description of the investment process. One aspect of the game that may seem peculiar is that, consistent with the usual formulation of agency models, the entire outcome is planned by the principal (the investor) subject to a reservation utility level for the agent (the manager). This is a useful device for mapping out the efficient frontier of contracts (as we vary the reservation utility level), and it gives the same set of efficient contracts as if plans were made by the manager or if the contract were the outcome of an efficient bargaining process. Another aspect of the game that may seem peculiar is that the portfolio manager does not actually choose the portfolio and instead the manager announces the information gathered and the portfolio choice is implemented by an accountant or computer according to a rule that is part of the contract. Again, this is not to be taken literally but is a good device (according to the revelation principle, discussed further below) to ensure we have the optimal contracting outcome. An equivalent but perhaps more palatable mechanism would have the manager making portfolio choices for the investor and manager subject to the constraint that the strategies have to be consistent with the contractual plan for some signal state.

The formal game has three stages. In the first stage, the contract is selected by the investor after which the manager either rejects it (and receives the reservation utility as the game ends) or accepts it. In the second stage, the manager chooses effort $a$ and receives a signal $s$ reported using the strategy $s^R(s)$, and the reported signal determines the portfolio choice as specified in the contract. In the final stage, the investment outcome is realized and the investor and manager are paid off. Formally, the game is chosen by having the investor choose the contract and intended effort level subject to a participation constraint (the manager gets at least the reservation utility level), and incentive compatibility (the manager has an incentive to expend

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8The model we describe in this paper is a one-period model in the sense that payoffs will be realized only once. Trading may take place at any number of dates in between the contracting date and the date at which the parties to the contract receive their payouts. So the investment strategy need not be a static portfolio. In particular infinitely many trading dates are allowed and the portfolio strategy chosen may be a dynamic strategy.
the intended effort and report the state truthfully).

Let the initial value of the portfolio be \( w_0 \) and the final value be denoted \( w_0 R_p \). The portfolio return \( R_p \) will depend not only on the market state but also on the the investment strategy followed and therefore on \( s^R(s) \). An equilibrium is a division of this final payoff into a portion paid to the manager and the remainder going to the investor. One simple assumption which could be made is that the manager’s payoff is a fixed fraction of the portfolio payoff or perhaps some nonlinear function of the portfolio payoff\(^9\). A slight generalization might be that the manager’s payoff is linear in \( R_p \) and the return on a benchmark portfolio. However it is not obvious that such restrictions would not be binding in the space of all contracts. Even assuming that the manager’s payoff is some function of these two returns is still restrictive. Why not some nonlinear function of the portfolio return and two benchmarks? or five? Also why restrict the payoff to depend on the reported signal only through the portfolio return? We wish to choose the most general contract possible.

We are aided in this endeavor by a concept from mechanism design called the revelation principle. In any of the mechanisms listed above (and any other mechanism) if there is an equilibrium then the allocations to investor and manager will depend on both the reported signal and the market state. Let \( \hat{\phi}(s^R(s), \omega) \) and \( \hat{V}(s^R(s), \omega) \) be payoff functions to manager and investor respectively which arise in some equilibrium of the contracting game. Now define

\[
\phi(s, \omega) \equiv \hat{\phi}(s^R(s), \omega)
\]

and

\[
V(s, \omega) \equiv \hat{V}(s^R(s), \omega).
\]

These payoff functions define the same equilibrium allocations and the manager will have no incentive to misreport the signal. This is called a direct mechanism. The revelation principle tells us that restricting attention to direct mechanisms is without loss of generality. Hence the most general contract solves

Problem 1 Choose the fee $\phi(s, \omega)$, investor payoff $V(s, \omega)$, and effort action $a$ to maximize the client’s expected utility

$$\int\int U^P(V(s, \omega))f(s, \omega; a)d\omega ds$$

subject to the manager’s participation constraint\(^{10}\)

$$\int\int U^A(\phi(s, \omega))f(s, \omega; a)d\omega ds - c(a) = u_0,$$

the budget constraint

$$\forall s \int (V(s, \omega) + \phi(s, \omega))p(\omega)d\omega = w_0,$$

and incentive compatibility, namely that $a' = a$ and $s^R(\cdot) = I(\cdot)$ maximize

$$\int\int U^A(\phi(s^R(s), \omega))f(s, \omega; a')d\omega ds - c(a'),$$

where $U^P$ and $U^A$ denote the investor’s and manager’s von Neumann-Morgenstern utility function for end-of-period wealth respectively. The manager’s utility cost of effort is $c(a)$ defined on $[0, \pi]$ for some $\pi \in (0, 1)$. The cost function $c$ is strictly increasing and strictly convex and satisfies $c'(0) = 0$ and $c'(\pi) = \infty$. The cost function and utility functions are assumed to be twice continuously differentiable.

Note that $\phi(s, \omega)$ and $V(s, \omega)$ have the familiar form from games of asymmetric information, namely that they define a menu of state contingent contracts, indexed by $s$. After the manager observes the signal it is communicated to the investor by the manager’s choice of contract. However note that since

$$w_0R_p = \phi(s, \omega) + V(s, \omega),$$

\(^{10}\)In principle, this could be an inequality constraint. This is without loss of generality when the utility function is unbounded below (as in our log utility examples). This could matter if there is a limit to how much an manager can be punished (as when there is no indentured servitude or debtor’s prison). Obviously, these are interesting cases, but they are just not the focus of this paper.
a choice of contract completely specifies the portfolio payoff as a function of the market state. Since markets are complete there is a portfolio strategy that delivers a given payoff function. Hence this equilibrium specifies a menu of portfolio strategies from which the manager can choose.

Therefore, to implement an optimal contract the investor must restrict the set of strategies available to the manager. Typically examinations of the performance evaluation question in the literature do not restrict how the manager achieves superior performance. Actual investment guidelines, on the other hand, are full of such restrictions. Common restrictions for asset allocation include restrictions on the universe of assets and ranges for proportions in the various assets; while common restrictions for management within an asset class are limitations on market capitalization or style (growth versus income) of stocks, credit ratings or durations of bonds, restrictions on use of derivatives, maximum allocations to a stock or industry, and increasingly portfolio risk measures such as duration, beta, or tracking error.

A Transformed and Specialized Problem

To be able to solve Problem 1, we need more structure. To begin with, we shall be more explicit about the effect of manager effort on the joint distribution of signal and market state.

Let \( f^U(s, \omega) \) and \( f^I(s, \omega) \) be two joint densities\(^{11}\) of \( \omega \) and \( s \) which we refer to as the uninformative and the informative densities respectively. We assume that both joint densities have the same support and further that the marginal distributions \( f^s(s) \) of \( s \) and \( f^\omega(\omega) \) of \( \omega \) are the same in both cases. If the manager expends effort \( a \in [0, 1] \) then the joint distribution will be the mixture distribution

\[
(6) \quad a f^I(s, \omega) + (1 - a) f^U(s, \omega).
\]

The effort level \( a \) can be interpreted as the probability of getting a signal drawn from the informative joint density \( f^I(s, \omega) \) instead of the uninformative density \( f^U(s, \omega) \). We assume without loss of generality that under the informative density \( s \) and \( \omega \) are positively correlated. We let \( f^U(s, \omega) = f^s(s) f^\omega(\omega) \) so that uninformative signals are independent of market states (hence the term “uninformative”). So we can write the joint density

\[^{11}\text{The densities can accommodate discrete state spaces by having underlying domains with discrete measure instead of Lebesgue measure.}\]
as

\[ f^{\omega}(\omega) + a(f^I(\omega|s) - f^{\omega}(\omega)) f^s(s) \]

It turns out that it will be convenient in what follows to use transformed choice variables as in Grossman and Hart (1983). If we define

\[ u^A(s, \omega) \equiv U^A(\phi(s, \omega)) \]

\[ u^P(s, \omega) \equiv U^P(V(s, \omega)) \]

we can rewrite Problem 1 above in an equivalent form in which the choice variables are \( u^A(s, \omega) \) and \( u^P(s, \omega) \). To write the budget constraint we must also define inverse utility functions \( I^A \) and \( I^P \) and then

\[ \phi(s, \omega) = I^A(u^A(s, \omega)) \]

and

\[ V(s, \omega) = I^P(u^P(s, \omega)). \]

We assume that both investor and manager have log utility so that \( I^A(x) = I^P(x) = \exp(x) \). One way in which this simplifies the problem above is that it allows us to reduce both the number of choice variables and the number of constraints. To do this we take the manager’s utilities as given and maximize the objective subject to the budget constraint to obtain the investor’s indirect utility as

\[ u^P(s, \omega) = \log \left( \frac{B^P(s) f^{\omega}(\omega) + a(f^I(\omega|s) - f^{\omega}(\omega))}{p(\omega)} \right) \]

where

\[ B^P(s) = w_0 - \int \exp(u^A(s, \omega)) p(\omega) d\omega \]

is the investor’s budget share. Equation (12) can be taken to be an application of the usual formula for optimal consumption given log utility and complete markets (in this case conditional on \( s \)).

The investor’s expected utility can now be computed as
(14) \[
\int \log \left( w_0 - \int \exp(u^A(s, \omega))p(\omega)d\omega \right) f^*(s)ds
\]
\[+
\int\int \log \left( \frac{af^I(\omega|s) + (1-a)f^U(\omega)}{p(\omega)} \right) (af^I(s, \omega) + (1-a)f^U(s, \omega))d\omega \]

Note that the second term depends only on effort, \(a\), and not on the manager’s utilities. This means we can ignore this term when solving the problem of what contract will implement a particular effort level and take it into consideration only when optimizing over effort levels. Note also that the first term is concave in the manager’s utilities.

The most difficult constraint to handle in Problem 1 is the incentive compatibility constraint expressed in equation (4). We can simplify this somewhat by examining the manager’s problem

**Problem 2** Choose \(a\) and \(s^R(\cdot)\) to maximize

(15) \[
\int\int u^A(s^R(s), \omega)(f^I(s, \omega) + a(f^I(s, \omega) - f^U(s, \omega)))d\omega = c(a).
\]

The first-order conditions of this problem are

(16) \[
\int\int u^A(s^R(s), \omega)(f^I(s, \omega) - f^U(s, \omega))d\omega = c'(a)
\]

and

(17) \[
(\forall s) \int \frac{\partial u^A(s^R(s), \omega)}{\partial s^R}(f^U(s, \omega) + a(f^I(s, \omega) - f^U(s, \omega)))d\omega = 0
\]

Since these must hold for any solution to this problem we may be able to use them in place of the incentive compatibility constraint. Using the first-order conditions for the manager’s problem in place of the incentive compatibility constraints in the main problem is called the first-order approach.

With these assumptions Problem 1 can be rewritten as
**Problem 3** Choose \( u^A(s, \omega) \) and \( a \) to maximize

\[
\int \log \left( w_0 - \int \exp(u^A(s, \omega))p(\omega)d\omega \right) f^s(s)ds 
\]

\[
+ \int \int \log \left( \frac{af^I(\omega|s) + (1-a)f^\omega(\omega)}{p(\omega)} \right) (af^I(s, \omega) + (1-a)f^U(s, \omega))dsd\omega 
\]

subject to the participation constraint

\[
\int \int u^A(s, \omega)(af^I(\omega|s) + (1-a)f^\omega(\omega))f^s(s)d\omega ds - c(a) = u_0, 
\]

incentive compatibility of effort

\[
\int \int u^A(s, \omega)(f^I(\omega|s) - f^\omega(\omega))f^s(s)d\omega ds = c'(a), 
\]

and incentive compatibility of signal reporting

\[
(\forall s) \int \frac{\partial u^A(s, \omega)}{\partial s}(af^I(\omega|s) + (1-a)f^\omega(\omega))d\omega = 0. 
\]

In a first-best world, the manager’s effort choice is contractible. The first-best contract is the solution to Problem 3 without imposing the incentive compatibility (IC) constraints (20) and (21). If we impose the IC-of-effort constraint but not the signal-reporting constraint, then we get a contract which we call second-best. In this case, we imagine that effort is not observable but the information obtained by the manager is observable by the investor so a punishment can be arranged so it cannot be misreported. We call the solution to Problem 3 as stated with both IC constraints the third-best contract.

Solving the problem using the first-order approach is not in general equivalent to solving the original problem. However, the two are equivalent in the first-best and in the second-best, because \( f^U(s, \omega) + a(f^I(s, \omega) - f^U(s, \omega)) \) is linear in \( a \) and \( c(a) \) is convex, implying that the first-best and second-best optimizations are convex problems\(^{12}\). Unfortunately we have no such result

for the truth telling constraint. For some of our numerical results we will use
the first-order approach (with (17)) to the third-best, which should be valid
if \( c(a) \) is sufficiently convex. In section V we have a numerical example in
which the first-order approach is not valid and we obtain a solution with a
“remotely” binding incentive compatibility constraint.

Solution of this problem can proceed in two stages. In the first stage we
solve the above problem for a fixed \( a \) to find the contract which will induce
the manager to take that effort level. Then we can search over \( a \) to find the
optimal effort level. Below we shall show the solution to the first stage for
the first-best, second-best, and third-best cases.

III  Numerical Results

Before turning to what we can learn analytically from these models, let us
consider some numerical results. These are based on analytical solutions
given in the later sections for the first-best and second-best problems, and
numerical optimization for the third-best.

The numerical results take \( f^I, f^U, \) and \( p \) as given. We could also take
\( c \) as given, but we find it more convenient to take \( a \) as given and leave \( c \)
implicit. This allows us to compare the functional forms of the first-best,
second-best, and third-best contracts simply. Choosing \( a \) optimally in each
case would make this comparison more difficult since in general a different
level of effort will be optimal in a first-best world versus a second-best world
versus a third-best world. By specifying \( a \) exogenously we can attribute the
differences between the contracts solely to the addition of the IC constraints.

For the “informed” and “uninformed” joint density of \( \omega \) and \( s \), we assume
joint normality with the same marginals and with and without correlation
\( \rho > 0 \). We think of this as a model of market timing, with \( \omega \) representing
the demeaned log market return in the usual lognormal model over one year.
Then,

\[
(22) \quad f^s(s) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{s^2}{2\sigma^2} \right),
\]

\[
(23) \quad f^\omega(\omega) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{\omega^2}{2\sigma^2} \right),
\]
(24) \[ f^I(\omega, s) = \frac{1}{2\sigma^2\pi \sqrt{1-\rho^2}} \exp\left(-\frac{(\omega^2 - 2\rho \omega s + s^2)}{2\sigma^2(1-\rho^2)}\right), \]

and

(25) \[ f^I(\omega|s) = \frac{1}{\sigma \sqrt{2\pi(1-\rho^2)}} \exp\left(-\frac{(\omega - \rho s)^2}{2\sigma^2(1-\rho^2)}\right). \]

And, state prices are consistent with Black-Scholes and can be computed as the discount factor times the risk-neutral probabilities.

(26) \[ p(\omega) = e^{-r} \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(\omega + \mu - r)^2}{2\sigma^2}\right) \]

In these expressions, \( r \) is the riskfree rate, \( \mu \) is the mean return on the market, \( \sigma \) is the standard deviation of the market return. Without loss of generality, the signal \( s \) has mean 0 and the same variance as the log of the market return.

We work with discretized versions of \( f^I, f^U \), and \( p \) with \( N \) market states and \( M \) signal states. In order to circumvent the difficulty imposed by the presence of \( \lambda'(s) \) in this first order condition of the third-best problem we work with a discrete version in which the reporting constraint is replaced by two sets of reporting constraints. The first set imposes the restriction that reporting the state just higher than the true state is not optimal and the other does the same for reporting the state just lower than the true state. Together this makes \( 2(M - 1) \) constraints. As the discretization becomes very fine this problem approximates the continuous state case.

The manager’s utilities from the first best problem are plotted in Figure 1. The parameters used are \( \mu = 0.15, \sigma = 0.2, \rho = 0.5, r = 0.05 \), and \( w_0 = 100 \). We chose a cost of effort function such that \( u_0 - c(a) = 0.955 \) and \( c'(a) = 0.2 \) at \( a = 0.5 \).

A visual inspection of the solution to the second and third-best contracts at these parameter values is not very instructive. However we can gain insight by examining the incremental changes in the contract when we move from first-best to second-best to third-best. Figure 2 plots the manager’s utilities in the second-best minus the manager’s utilities in the first-best. When signal and market are both high (or both low), \( f^I(\omega|s) > f^U(\omega) \), so the manager is rewarded in those states. In the other corners of the distribution, the manager has less utility than in the first-best case. This provides the incentive to exert effort.
Figure 1: First-best Contract
Figure 2: Second-best minus First-best Utility Levels
Figure 3: Third-best minus Second-best Utility Levels

Figure 3 plots the manager’s utilities in the third-best minus the manager’s utilities in the second-best. The difference between these two contracts is that the third-best provides incentives to truthfully report. An manager facing a second-best contract would tend to be overly conservative in reporting to reduce the extra risk exposure the second-best contract offers compared to the first best. From the figure, we can see that compared to the second-best an manager reporting an extreme signal has a higher payoff and less risk.

If we choose a higher value of $c'(a)$ we obtain a forcing contract as in Figure 4. Note the punishments in the corner states. Just as in the smooth case the second-best contract is not third-best. Because the punishments are concentrated in one state the manager would just misreport that state as being the one next to it. To make this contract third-best the punishment must be spread out across signal states in the extreme market states to maintain truthful reporting. This is illustrated in figure 5. This figure shows the numerical solution to the same problem from Figure 4 with the addition
Figure 4: Second Best: Forcing Case

of truthful reporting constraints.

IV Analytical Results

The First-best Contract

The first-best contract solves Problem 3 neglecting the incentive compatibility constraints (20) and (21). In a first-best contract we expect to find that there is optimal risk sharing between the manager and the investor. This means that the marginal utility of wealth for the manager should be proportional to the investor’s marginal utility in all states.

The first-order condition for $u^A$ is

$$\frac{\exp(u^A(s,\omega))p(\omega)}{B^P(s)} = \lambda_R(f^\omega(\omega) + a(f^I(\omega|s) - f^\omega(\omega)))$$
Figure 5: Third Best: Forcing Case
where $\lambda_R$ is the Lagrange multiplier on the participation constraint. Multiplying both sides by $B^P(s)$ and integrating both sides with respect to $\omega$ we obtain

$$B^A(s) = \lambda_R B^P(s).$$

(28)

Substituting this into the budget constraint we have that

$$B^P(s) = \frac{w_0}{1 + \lambda_R}$$

(29)

from which we obtain

$$u^A(s, \omega) = \log \left( \frac{w_0 \lambda_R}{(1 + \lambda_R)} \frac{f^\omega(\omega) + a(f^I(\omega|s) - f^\omega(\omega))}{p(\omega)} \right).$$

(30)

Comparing this with equation (12), substituting the definition of $B^P(s)$ from above, we see that the first-best contract with log utility is a sharing rule which gives the manager a fixed proportion of the payoff of the portfolio independent of the signal. So, as expected, optimal risk sharing obtains.

The portfolio payoff in this case has an interesting interpretation. Suppose that rather than hiring the manager the investor were to manage the portfolio but without the benefit of the information gathering efforts of the manager. The payoff to the investor would be $w_0 R^B$ where

$$R^B = \frac{f^\omega(\omega)}{p(\omega)}.\quad (31)$$

Since this portfolio involves no superior information and because it would be the investor’s optimal portfolio in the absence of the manager we call it the “benchmark” portfolio. Similarly define

$$R^I = \frac{f^I(\omega|s)}{p(\omega)}\quad (32)$$

which is the gross return$^{13}$ on a portfolio which depends on the manager’s signal. The return on the total portfolio is equal to the benchmark return plus a constant times the excess return of some “informed portfolio” over the

$^{13}$To see that this is a gross return note that if we multiply by state prices and integrate over $\omega$ we would get unity because the numerator is a probability density. Since markets are complete there is a portfolio that has this payoff.

22
benchmark. The investor and manager each get a proportional share of the portfolio payoff. Formally,

\[ R_p = R^B + a(R^I - R^B), \]

(33)

\[ V(s, \omega) = B^P R_p, \]

(34)

\[ \phi(s, \omega) = B^A R_p. \]

(35)

**The Second-best Contract**

To obtain the second-best contract we solve the same problem as in the first-best case with the additional constraint (20) which states that the contract is incentive compatible for effort. The first-order condition for \( u^A(s, \omega) \) is

\[ \exp(u^A(s, \omega)p(\omega)) = \lambda_R(f^\omega(\omega) + a(f^I(\omega|s) - f^\omega(\omega))) + \lambda_a(f^I(\omega|s) - f^\omega(\omega)) \]

(36)

where \( \lambda_a \) is the Lagrange multiplier on the IC constraint. Proceeding as before we obtain

\[ u^A(s, \omega) = \log \left( \frac{w_0\lambda_R}{(1 + \lambda_R)} \frac{f^\omega(\omega) + (a + \frac{\lambda_a}{\lambda_R})(f^I(\omega|s) - f^\omega(\omega))}{p(\omega)} \right) \]

(37)

but the expression for the investor’s utility is the same as in the first-best case. Hence the second-best contract does not exhibit optimal risk sharing.

Equation (37) is very similar to (30) except that it has \( a + \lambda_a/\lambda_R \) rather than \( a \) multiplying the second term. We know \( \lambda_R \) will be positive because of the positive marginal utility of wealth. At the optimal \( a \) we will have \( \lambda_a \) positive as well. This means that the second-best contract gives the manager more exposure to the excess return portion of the contract. This is what gives the manager the incentive to expend effort \( a \). It seems crucial that the manager cannot misreport the signal; if the manager could do so, reporting a less extreme signal is likely going to undo (to some extent) this artificially high exposure and it may not be possible for the investor to impose useful incentives in the contract beyond what is optimal effort in the manager’s own portfolio.
Recalling the definitions of $R_B$ and $R_I$ we can write the return on the portfolio as

$$R_p = R_B + \left( a + \frac{\lambda a}{1 + \lambda R} \right) (R_I - R_B).$$

Now we can write $R_I$ as a function of $R_p$ and $R_B$ such that the manager’s fee can be written

$$\phi(s, \omega) = B^A (R_p + k(R_p - R_B))$$

where

$$k = \frac{\lambda a}{a + \frac{\lambda a}{1 + \lambda R}} \geq 0.$$  

This suggests the following intuitive decomposition into a performance measure $m = R_p - R^B$ and a fee schedule which is increasing in the performance measure, i.e. $B^A(R_p + km)$. When $k = 0$, as it would if the effort constraint did not bind, this reduces to the first-best manager payoff. The fact that the performance measure is the excess return of the portfolio over a benchmark has intuitive appeal. Measuring performance relative to a benchmark is common practice in the portfolio management industry.

There is another difference between this contract and the first-best contract which is not apparent at first glance. Notice that the first best contract is defined for all $a \in [0, 1]$. In general, the optimal $a$ can lie anywhere in that interval\(^{14}\) but the contract will be of the same form. For the second-best contract, we must have $a + \lambda a/\lambda R \in [0, 1]$. To see why, note that in (37) if $a + \lambda a/\lambda R > 1$ then the manager’s consumption will be negative in states for which $f^\omega(\omega) > f^I(\omega|s)$. These states represent times in which a low market state occurs when the manager’s signal would have predicted a high market state or the converse. But log utility goes to minus infinity at zero and therefore negative consumption with a positive probability is more than a large punishment and is in fact infeasible.

As $a + \lambda a/\lambda R$ increases (varying $\lambda R$ to maintain the participation constraint), the corresponding value of $c'(a)$ that will satisfy (20) increases. Denote by $c'$ the value of $c'(a)$ which satisfies (20) for $a + \lambda a/\lambda R = 1$, i.e.

$$c' \equiv \int \int \log \left( \frac{w_0 \lambda_R}{1 + \lambda R} \frac{f^I(\omega|s)}{p(\omega)} \right) \left( f^I(\omega|s) - f^\omega(\omega) \right) f^s(s) d\omega ds$$

\(^{14}\)Which level of effort is optimal for the problem will depend on $c, f^I, f^U, \text{and } p$. 

24
For effort levels which do not satisfy $c'(a) \leq \bar{c}$ the second-best can be approached by a sequence of contracts that look very much like the contract with $a + \lambda_a/\lambda_R = 1$ but with additional punishment in remotely possible states to obtain incentive compatible effort. Specifically, we have the following theorem.

**Theorem 1** When $c'(a) \leq \bar{c}$ the solution of the second-best problem is of the form (37). For $c'(a) > \bar{c}$ the second-best can be achieved in the limit as $n \uparrow \infty$ of a sequence of contracts of the form\(^{15}\)

\[ u_n^A(\omega, s) \equiv \nu_{0n} + \log \left( \frac{f^I(\omega|s)}{p(\omega)} \right) - \nu_{1n}(\omega < -n)(s > n) \tag{42} \]

for $n$ sufficiently large, where $\nu_{1n}$ is chosen to satisfy incentive compatibility of effort (20) and $\nu_{0n}$ is chosen to satisfy the participation constraint (19).

**Proof** See appendix

We call this a forcing contract but it differs from what is usually meant by this term. Mirrlees (1974) gave conditions under which the first-best solution to an agency problem can be approached by a sequence of contracts with larger and larger punishments in a smaller and smaller set of extreme states. Mirrlees’ forcing contract works when likelihood ratios become unbounded in extreme states. Note that the mixing form of the densities we have assumed precludes this happening: for two interior values of $a$, the ratio of their likelihoods is always bounded uniformly across $\omega$. For this case the punishments are chosen to maintain incentives so that each element in the sequence is feasible. The limiting contract gives the investor at least as much value as any feasible contract but is not itself feasible because it provides too little incentive for effort and gives too much utility to the manager. This can be interpreted as an $\epsilon$-equilibrium because you can get arbitrarily close to the second-best with some element along the sequence.

\(^{15}\)By $(s > n)$ we mean the indicator of that condition, i.e.,

\[ (s > n) \equiv \begin{cases} 1 & s > n \\ 0 & \text{otherwise} \end{cases} \]
The Third-best Contract

The third-best solves Problem 3 with all the constraints. The first-order condition for $u^A$ is

$$\frac{\exp(u^A(s, \omega))p(\omega)}{B^P(s)} = \lambda_R(f^\omega(\omega) + a(f^I(s) - f^\omega(\omega)))$$

$$+ \lambda_a(f^I(s) - f^\omega(\omega)) - a\lambda_s(s)\frac{\partial f^I(\omega|s)}{\partial s}$$

$$- \lambda'_a(s)(af^I(\omega|s) + (1 - a)f^\omega(\omega))$$

where $\lambda_s(s)$ is the Lagrange multiplier on the truthful reporting constraint. In this case we have

$$B^P(s) = \frac{w_0}{1 + \lambda_R - \frac{\lambda'_a(s)}{f^I(s)}}$$

and

$$B^A(s) = \frac{w_0(\lambda_R - \frac{\lambda'_a(s)}{f^I(s)})}{1 + \lambda_R - \frac{\lambda'_a(s)}{f^I(s)}}.$$

It does not seem possible to solve for $\lambda_a(s)$ analytically.

V A Remotely-binding Incentive Constraint

Now we turn to the case when the first-order approach may not be valid for the third-best. From problem 2 we saw that for any given reporting strategy the non-effort portion of manager expected utility is linear in $a$. Maximizing over reporting strategies gives the maximum of linear functions which is convex. The cost of effort is convex so the manager’s expected utility, viewed as a function of effort, is a convex function minus a convex function. If the curvature of the cost of effort were sufficient to overcome the convexity of the non-effort portion of expected utility then the problem would be concave and the first-order approach would be valid. If this is not the case then there is no guarantee that some combination of shirking and misreporting might not destroy the equilibrium suggested by a solution derived using the first-order approach.
To solve this problem in general we would need to add constraints of the following form: if the investor wants the manager to expend effort level $a^*$ then the manager cannot choose any $\hat{a} < a^*$ together with reporting strategy that will do better than choosing $a^*$ and truthfully reporting. By construction choosing $a^*$ and truthfully reporting will give the reservation utility. Therefore the IC of effort constraint in the problems of previous sections would need to be replaced by constraints of the form

$$\max_{s^{R}()} \int \int u^A(s^{R}(s), \omega)(\hat{a} f^I(\omega | s) + (1 - \hat{a}) f^\omega(\omega)) f^s(s)d\omega ds - c(\hat{a}) \leq u_0$$

for each $\hat{a} < a^*$. In general these constraints make the problem intractable.

However we can recover tractability by assuming that the cost of effort is linear\(^{16}\) up to some $\bar{a} \leq 1$ and infinite\(^{17}\) thereafter. Since the non-effort portion of expected utility is also linear in $\hat{a}$ for any choice of $s^{R}(s)$ there are only two possible levels of effort that can be optimal for the manager: zero effort and $a = \bar{a}$. There are three possible equilibria. In the first, zero effort is optimal and allocations are first-best. In the second $\bar{a}$ is optimal but incentive pay is not necessary and again allocations are first-best. In the third $\bar{a}$ is optimal but the manager must be given incentives to expend this effort. In this case there is only one level of shirking, $a = 0$, so we need only one constraint of the form given in equation (46). However at $a = 0$ the manager knows that the signal is completely uninformative so the optimal reporting strategy will not depend on the signal. Instead the manager will disregard the signal and report some $\hat{s}$. Therefore we can replace (46) with

$$\forall \hat{s} \int u^A(\hat{s}, \omega) f^\omega(\omega)d\omega \leq u_0.$$  

On intuitive grounds we can guess something about the behavior of the solution with this new set of constraints applied. Imagine that the manager decides to expend no effort. What signal would be the best to report? To make it more concrete imagine that the manager is a market timer and the reported signal determines the position in the market and cash. Would

\(^{16}\)This implies that the cost of $a = 0$ is zero.

\(^{17}\)Actually it wouldn’t have to be infinite for effort levels above $\bar{a}$. All that is needed is that it be large enough such that choosing an effort level this high would never be optimal.
the manager want to report a signal that prescribes holding no cash and buying the market on margin? or taking large short positions in the market portfolio? Neither seems likely since the probability that either of these extreme positions could be disastrous for the fund is quite large. Hence the manager will always want to report a state near the middle. So we expect there will be an interval near the mean of $s$ where the above constraints will bind but we expect the constraints for extreme values of $s$ not to bind. The effect of these binding constraints will be to take away utility in the states where the constraints bind to keep a shirking manager from being able to benefit by reporting these states. Notice that this is qualitatively similar to the contract feature we noticed earlier which gave incentives to report more extreme signals. We interpreted this as incentives to discourage “closet indexing”. Previously this incentive was provided by the truthful reporting constraints. Here it is provided by the constraints in (47).

Of course, taking away utility in some states and not in others gives an incentive to report a signal just outside this region. The truthful reporting constraints will fix this by taking away utility in these signal states if the manager is incorrect or adding utility in these states where the manager is correct. A shirking manager will still not find it attractive to report these states because of the punishment for being wrong.

In figure 6 we plot the utilities for the manager in this model minus the utilities in the first-best model. We see the effects of both sets of IC constraints. Note that compared with the first-best there is less utility in the middle states. Just to the side of this region there is a payoff which gives more utility to managers whose signal is correct which is what we would expect.

VI Conclusion

We have proposed a new model of optimal contracting in the agency problem in delegated portfolio management. We have shown that in a first-best world with log utility the optimal contract is a linear sharing rule over a portfolio which is equal to a benchmark portfolio plus an excess return of a portfolio which depends on the manager’s signal. In a second-best world the contract is of the same form except that the manager’s payout is more tilted toward the excess return to give incentives to the manager to work hard. We have also shown numerical results for the third-best case, which is close
Figure 6: Third-best minus First-best Manager Utilities for linear cost of effort
to the second-best in the examples we have examined. These results have been demonstrated in the context of a realistic return model and the derived performance measurement criterion looks more like the simple benchmark comparisons used by practitioners than more elaborate measures such as the Jensen measure, Sharpe measure, or various marginal-utility weighted measures. In addition, the optimal contract includes restrictions on the set of permitted strategies and also includes prior communication of information. These institutional features are more similar to practice than other existing agency models in finance.

We have only just started to tap the potential of this framework to tell us about agency problems in portfolio management. Although some of the general results extend to stock selection models as well as the market timing examples given in this paper, it would be interesting to see the exact form of the contracts for stock-pickers. Analyzing career concerns would be an interesting variant: in this case, the current client has to take as given the manager’s incentives to demonstrate superior performance this period in order to attract new clients or achieve a larger wage next period. In this case, there is probably a limit to the extent to which the client can neutralize the impact of career concerns. It would also be interesting to consider problems in which the manager’s utility function (as well as consumption) is bounded below, given that the actual economy has restrictions on indentured servitude. Rajan and Srivastava (2000) considers a simple model of delegated portfolio management with limited punishment. It would be interesting to see what limited punishment or career concerns would imply in our model.

In the model, we have obtained a lot of mileage from the transparent and frictionless markets assumption that allows us to look at an equivalent formulation in which the manager simply reports information and does not actually manage the money. However, there are aspects of performance (such as quality of execution) that are not handled adequately in this way. While institutions receive complete reports of which trades were made (and mutual fund performance reports can depend in this information in any necessary way as computed credibly by the custodian or consultant), even the full trade record combined with full quote and trade histories of each stock would not necessarily tell us what trading opportunities were available at each point in time. It would be useful to have a fuller exploration of when the reporting formulation is equivalent and of what happens otherwise. Another extension would include explicitly the two levels of portfolio management we see in practice, with the separation of responsibilities for asset allocation across
asset classes and management of sub-portfolios in each asset class. The ultimate beneficiaries have to create incentives for the overall manager to hire and compensate the asset class managers, and this could be modeled as a hierarchy of agency contracts.
Proof of Theorem 1

Proof. The text already explained why the second-best is achieved by a first-order solution of the form (37) when \( c'(a) \leq \bar{c} \). For \( c'(a) > \bar{c} \), let

\[
M^* = c'(a) - \int \int \log \left( \frac{f_I(\omega|s)}{p(\omega)} \right) (f_I(\omega|s) - f^*(\omega)) f^*(s) d\omega ds,
\]

which is positive by definition of \( \bar{c} \), and let

\[
u_{1n} = M^* - \int_{\omega<-n} \int_{s>n} (f_I(\omega|s) - f^*(\omega)) f^*(s) d\omega ds
\]

and

\[
u_{0n} = u^* + M^* \frac{\int_{\omega<-n} \int_{s>n} (a f_I(\omega|s) + (1-a) f^*(\omega)) f^*(s) d\omega ds}{\int_{\omega<-n} \int_{s>n} (f_I(\omega|s) - f^*(\omega)) f^*(s) d\omega ds}
\]

\[
\xrightarrow{n \uparrow \infty} u^* + M^*(1 - a),
\]

since \( f_I(\omega|s)/f^*(\omega) \to 0 \) uniformly for \( \omega < -n \) and \( s > n \). Now consider the pointwise limit of \( u_n^A(\omega, s) \), which is

\[
u_{1n} = \frac{M^*}{\int_{\omega<-n} \int_{s>n} (f_I(\omega|s) - f^*(\omega)) f^*(s) d\omega ds}
\]

and

\[
u_{0n} = u^* + M^*(1 - a) + \log \left( \frac{f_I(\omega|s)}{p(\omega)} \right).
\]

It is not hard to show that there is uniform convergence of the corresponding consumption \( \exp(u_n^A(\omega, s)) \to \exp(u_0^A(\omega, s)) \) since the \( \nu_1 \) term reduces consumption towards zero only in states for which consumption was already very small. Consequently the investor’s utility also converges to that corresponding to \( u_n^A(\omega, s) \) as \( n \uparrow \infty \). Note that \( u_0^A \) itself is not feasible: it gives too little incentive for effort and too much utility to the manager.

We have left to show that \( u_0^A \), which has the same value for the investor as the limit of the sequence \( u_n^A \), gives the investor at least as much value as any feasible \( u^A \). Let \( EU^F(u^A) \) represent the investor’s expected utility
as computed by (12) given an manager’s utility pattern  \( u^A \). Then, we can use concavity of  \( Eu^P \) to bound the value for any feasible  \( u^A \) by taking the derivative at  \( u^*_A \):

\[
(53) \quad Eu^P(u^A) = \int \log \left( w_0 - \int p(\omega) \exp(u^A(\omega, s)) d\omega \right) f^s(s) ds \\
\leq \int \log \left( w_0 - \int p(\omega) \exp(u^*_A(\omega, s)) d\omega \right) f^s(s) ds \\
- \int \int \frac{p(\omega) \exp(u^*_A(\omega, s))}{w_0 - \int p(\omega) \exp(u^*_A(\omega, s)) d\omega} (u^A(\omega, s) - u^*_A(\omega, s)) f^s(s) d\omega ds \\
= Eu^P(u^*_A) - \frac{\exp(u^* + M^*(1-a))}{w_0 - \exp(u^* + M^*(1-a))} \\
\times \int \int (u^A(\omega, s) - u^*_A(\omega, s)) f^I(\omega|s) f^s(s) d\omega ds
\]

It remains to show that the integral in the last right-hand expression is zero. Equations (19), (49), (52), and the fact that densities integrate to 1 imply that

\[
(54) \quad \int \int (u^A(\omega, s) - u^*_A(\omega, s))(af^I(\omega|s) + (1-a)f^s(\omega)) f^s(s) d\omega ds = M^*(1-a)
\]

and (20), (48), (52), and the fact that densities integrate to 1 imply that

\[
(55) \quad \int \int (u^A(\omega, s) - u^*_A(\omega, s))(f^I(\omega|s) - f^s(\omega)) f^s(s) d\omega ds = M^*.
\]

Adding (54) and (1-a) times (55), we get

\[
(56) \quad \int \int (u^A(\omega, s) - u^*_A(\omega, s)) f^I(\omega|s) f^s(s) d\omega ds = 0,
\]

as was required to complete the proof.
References


