Collected Works of Stephen A. Ross: Some Highlights

Philip H. Dybvig
Washington University in Saint Louis

西南财经大学 (Xīnán Cái jīng Dà xué)
December 3, 2008
Professor Stephen A. Ross, MIT Sloan School

• Franco Modigliani Professor Finance and Economics, 1998–present
• Yale University, 1977–1998
• University of Pennsylvania, 1970–1978
• PhD (Harvard Economics) 1969
• BS (CalTech Physics) 1965
• born February 3, 1944

Chester Spatt and I are editing Steve’s collected works. Chester and I are both students and co-authors of Steve’s. I was also Steve’s colleague at Yale for seven years.
Papers

Steve is the greatest living finance scholar. Steve’s vitae lists more than 100 papers, most of which are published in top journals. These publications cover many areas, including:

- Agency Theory
- APT
- CAPM
- Fixed Income
- Growth Theory
- Industrial Organization
- Insurance
- International Economics
- Investments
- Labor Economics
- Option Pricing
- Real Estate
- Signalling
- Survivorship Bias
- Taxes
- and more.

Steve’s works also include some papers that cross areas and are hard to classify, including interesting “big picture” topics such as Financial Marketing, Forensic Finance, and Behavioral Finance.
Some (but not all!) of Steve’s Pathbreaking Papers

Ross (AER 1973) - Agency Theory
Ross (JET 1976) - APT
Cox and Ross (JFE 1976) - Risk-neutral pricing
Ross (QJE 1976) - spanning with options
Ross (BellJ 1977) - Signalling and Capital Structure
Ross (JB 1978) - Arbitrage and Linear Pricing Rule (Fundamental Theorem of Asset Pricing)
Cox, Ross, Rubinstein (JFE 1979) - Binomial Option Pricing Model
Cox, Ingersoll, Ross (JF 1981) - Expectations Hypothesis
Cox, Ingersoll, Ross (Econometrica 1985a,b) - CIR model
Gibbons, Ross, Shanken (Econometrica 1989) - maximum-likelihood CAPM test
Agency Theory


This paper introduced agency theory to economics and finance. It is a very short paper but very important. The method for analyzing agency problems is still widely used today.

In the law, an agency relationship is one in which one person (the principal) hires another person (the agent) to perform some task. Because the principal and agent have different preferences about unobservable actions (for example over effort), there is a fundamental conflict that does not arise in the standard competitive model (in which all conflicts are internalized through the price system).
The Principal’s Problem

Choose a fee schedule $\phi(Y)$ and the agent’s optimal effort $e^*$ to maximize

$$EU_P(Y(e^*, \varepsilon) - \phi(Y(e^*, \varepsilon)))$$

subject to the agent’s participation constraint:

(i) $EU_A(\phi(Y(e^*, \varepsilon))) \geq U_R$

and incentive-compatibility of effort:

(ii) choosing $e = e^*$ maximizes $EU_A(\phi(Y(e, \varepsilon)))$. 
Main Results

- Trade-off between incentives and risk-sharing
- If the agent is risk-neutral, the first-best is available

The trade-off between incentives and risk sharing is the normal case. For optimal incentives, the agent has a big exposure to the risk of the outcome. For optimal risk-sharing, the principal shares more of the risk.

As an exception, if the agent is risk-neutral, having the agent bear all the risk fills both needs. Specifically, in this case we choose \( e^* \) that maximizes \( EY(e, \varepsilon) \) and then we choose the fee schedule \( \phi(Y) = Y - k \) where \( k = EY(e^*, \varepsilon) - U_R \). Because the same contract gives first-best incentives and first-best risk-sharing, there is no trade-off in this case.
Really Risk-neutral

Here, risk-neutral means really risk neutral (linear von Neumann-Morgenstern preferences over all wealth levels, not just positive wealth levels). Really risk neutral is equivalent to being indifferent between any gamble and its mean payoff. In recent years, it is common to call an agent with linear preferences risk neutral, even if consumption is only defined over positive wealth or some other restricted domain. I guess this started as a sloppy misapplication of the equivalence; this change in terminology might lead to a misunderstanding of Steve’s result.
Arbitrage Pricing Theory (APT) Papers


APT Assumptions and SML

The Arbitrage Pricing Theory (APT) extends the CAPM to a theoretical model pricing multiple factors. Suppose risky asset returns are given by a factor structure

\[ x_i = \mu_i + \sum_{k=1}^{K} \beta_{ik} f_k + \varepsilon_i, \]

where \( \mu_i \) is the mean return on asset \( i \), \( f_k \) is a random mean-zero factor payoff, and \( \beta_{ik} \) is a constant giving the loading of asset \( i \) on factor \( k \), and \( \varepsilon_i \) is a mean-zero error term uncorrelated across assets. Then the APT asserts that there exists a shadow riskfree rate \( r^f \) (equal to the actual riskfree rate if there is a riskfree asset) and factor risk premia \( \lambda_k \) such that, for all \( i \),

\[ \mu_i = r^f + \sum_{k=1}^{K} \beta_{ik} \pi_k. \]

This is the APT’s Security Market Line (SML).
Properties of the CAPM and APT

• Market-level risk is priced (CAPM and APT)
• Idiosyncratic risk is not priced (CAPM and APT)
• Diversification pays (CAPM and APT)
• single source of priced risk, implying 2-fund separation (CAPM or 1-factor APT)
• multiple sources of priced risk, implying $K$-fund separation ($K$-factor APT)

The “market model” version of the CAPM is formally subsumed by the 1-factor APT, and in fact Steve has argued persuasively that most tests of the CAPM implicitly use the one-factor APT as the null hypothesis.
Origins of the APT: mutual fund separation

Steve (JET 1978) showed that having a factor structure for returns is a sufficient condition for $K$-fund separation (i.e., all risk averse agents would be happy to hold portfolios of the $K$ “mutual funds” instead of a general portfolio). Specifically, if the vector of asset payoffs per dollar invested in a one-period model are given by $x = (x_1, \ldots, x_N)$, then a sufficient condition for $K$-fund separation is the existence of portfolios $\theta_1, \ldots, \theta_K$, such that, for each $n$, there exists weights $w_{ik}$ such that

$$x_n = \sum_{k=1}^{K} w_{ik} \theta_k + \varepsilon_i$$

where $E[\varepsilon_i | \theta_1, \ldots, \theta_K] = 0$. This is very close to the APT. Steve’s paper shows that this characterization is necessary as well for 2-fund separation, and shows the route for proving necessity for $K$-fund separation.
Absence of Arbitrage and the Linear Pricing Rule


Steve has argued that absence of arbitrage is the unifying theme for all of finance. The results in these papers (plus the Friend-Bicksler volume piece op. cit.), are summarized in the Fundamental Theorem of Asset Pricing and the Pricing Rule Representation Theorem.
Fundamental Theorem of Asset Pricing

Theorem (Fundamental Theorem of Asset Pricing) The following are equivalent:
(i) Absence of arbitrage
(ii) Existence of a positive linear pricing rule that prices all claims
(iii) Existence of a hypothetical agent who prefers more to less and has an optimal portfolio

The equivalence of (i) and (ii) was the point of Steve’s landmark paper “A Simple Approach to the Valuation of Risky Streams.” That was the heavy lifting. Inventing the name and adding the third property (iii) is something I did when for teaching doctoral finance at Princeton. It is obvious that (iii) implies (i), which reminds students why we are interested in absence of arbitrage in the first place. The other direction is more subtle and very useful. For example, if someone asks whether our no-arbitrage result is consistent with equilibrium, the answer is yes by (i) implies (iii).
Some Technical Issues

The FTAP is true essentially as stated in a finite-dimensional context, as proven by Steve in the piece in the Friend and Bicksler volume. In infinite dimensions, things are more subtle and depend some on topological properties of the space, since this is an extension theorem. (Absence of arbitrage alone gives that the pricing rule is positive and unique on the marketed space, but topology is required to extend this to a positive linear pricing rule over “all” claims.) “A Simple Approach” gives a rigorous proof but makes a strong assumption that the interior of the positive orthant is nonempty, which does not seem to get us much beyond $L^\infty$. 
Choice of Topology

For definitions of asymptotic arbitrage on sequences, the third leg reminds us why we are interested in assuming no arbitrage in the first place and suggests using a topology in which preferences are continuous. Unfortunately, reasonable preferences we normally use are not continuous in any topology we use. For example, log and power utility are not defined on any neighborhood of initial consumption in $L^p$ for $1 \leq p < \infty$ if the state-space is infinite-dimensional (since the positive orthant has empty interior in this space), or for $L^\infty$ if consumption is not bounded below away from zero (and it typically isn’t, e.g. if the state-price density is unbounded above). This argument suggests that our usual choices of topology are not strong enough to assure that an approximate arbitrage sequence is attractive (or even feasible) far enough out the sequence for an agent with reasonable preferences.
Pricing Rule Representation Theorem

When arbitrage pricing goes to work, it is useful in different forms in different contexts.

Theorem (Pricing Rule Representation Theorem) The following are equivalent: (i) Existence of a positive linear pricing rule (ii) Existence of consistent risk-neutral probabilities (iii) Existence of a positive state-price density (iv) Existence of positive state prices

The most famous of these is (ii), which is often called the martingale approach. I think many people do not know that this approach originated in Steve’s work, especially Cox-Ross [1978]. The state-price density (iii) is also known as the stochastic discount factor or pricing kernel.
Using the various representations

Using general linearity is useful for proving a general distribution-free result such as put-call parity or Modigliani-Miller. The values add at maturity; linearity implies that values add now, too: \( S + P = L(S) + L((X - S)^+) = L(S + (X - S)^+) = L((S - X)^+ + X) = L((S - X)^+) + L(X) = C + B \). Using risk-neutral probabilities, (the martingale approach) is most useful for pure valuation problems for which risk pricing is an unnecessary complication. The state-price density, also known as the stochastic discount factor or pricing kernel, is especially good for portfolio problems because then reward for risk-taking does matter, and in fact the state-price density is proportional to the marginal utility of consumption in the standard problem. State prices are most useful for economic analysis, especially when there are state claims in discrete state spaces and agents have different subjective probabilities.
Arbitrage pricing: more examples

Using the martingale approach is often convenient for pure asset pricing problems:

\[ p_0 = E^*[ \frac{1}{1 + r} p_1 ] \]

Using the state-price density is good for consumption problems:
Choose \( c \) to
maximize \( Eu(c) \)
s.t. \( E[\rho c] = w_0 \)
foc: \( u'(c) = \lambda \rho \)
Conclusion

Steve Ross has had a great influence on research and practice in finance. We have had a look at only a few of his important papers.
Epilogue: some current working papers by Ross’s student

Theoretical Corporate: “Renegotiation-proof Contracting, Disclosure, and Incentives for Efficient Investment” with Nina Baranchuk (UT-Dallas) and Jun Yang (Indiana)

Empirical Corporate: “Money Grab in China,” joint with Yingxue Cao (Tsinghua) and Joseph Qiu (Temple)

Theoretical Investments: “Preference for Foreknowledge,” with Chris Rogers (Cambridge)

Portfolio Delegation: “Portfolio Performance and Agency” with Heber Farnsworth (NISA) and Jennifer Carpenter (NYU)

Insurance: “Optimal Casualty Insurance and Regulation in the Presence of a Securities Market,” with An Chen (Bonn)

http://dybfin.wustl.edu/research